

Physics Notes Class 11 CHAPTER 12

THERMODYNAMICS

The branch dealing with measurement of temperature is called thermometry and the devices used to measure temperature are called thermometers.

Heat

Heat is a form of energy called thermal energy which flows from a higher temperature body to a lower temperature body when they are placed in contact.

Heat or thermal energy of a body is the sum of kinetic energies of all its constituent particles, on account of translational, vibrational and rotational motion.

The SI unit of heat energy is joule (J).

The practical unit of heat energy is calorie.

$$1 \text{ cal} = 4.18 \text{ J}$$

1 calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C.

Mechanical energy or work (W) can be converted into heat (Q) by $1 \text{ W} = JQ$

where J = Joule's mechanical equivalent of heat.

J is a conversion factor (not a physical quantity) and its value is 4.186 J/cal.

Temperature

Temperature of a body is the degree of hotness or coldness of the body. A device which is used to measure the temperature, is called a thermometer.

Highest possible temperature achieved in laboratory is about 10⁸ while lowest possible temperature attained is 10⁻⁸ K.

Branch of Physics dealing with production and measurement temperature close to 0 K is known as cryogenics, while that dealing with the measurement of very high temperature is called pyrometry. Temperature of the core of the sun is 10⁷ K while that of its surface 6000 K.

NTP or STP implies 273.15 K (0°C = 32°F).

Different Scale of Temperature

1. **Celsius Scale** In this scale of temperature, the melting point of ice is taken as 0°C and the boiling point of water as 100°C and space between these two points is divided into 100 equal parts
2. **Fahrenheit Scale** In this scale of temperature, the melting point of ice is taken as 32°F and the boiling point of water as 212 and the space between these two points is divided into 180 equal parts.
3. **Kelvin Scale** In this scale of temperature, the melting point of ice is taken as 273 K and the boiling point of water as 373 K the space between these two points is divided into 100 equal parts

Relation between Different Scales of Temperatures

$$\frac{C}{100} = \frac{F - 32}{180} = \frac{K - 273}{100} = \frac{R}{80}$$

Thermometric Property

The property of an object which changes with temperature, is called thermometric property. Different thermometric properties thermometers have been given below

(i) Pressure of a Gas at Constant Volume

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

and

$$P_t = P_0 \left(1 + \frac{t}{273} \right)$$

$$t = \left(\frac{P_t - P_0}{P_{100} - P_0} \times 100 \right)^{\circ}\text{C}$$

where p , p_{100} , and p_t , are pressure of a gas at constant volume 0°C , 100°C and $t^{\circ}\text{C}$.

A constant volume gas thermometer can measure temperature from -200°C to 500°C .

(ii) Electrical Resistance of Metals

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where α and β are constants for a metal.

As β is too small therefore we can take

$$R_t = R_0(1 + \alpha t)$$

where, α = temperature coefficient of resistance and R_0 and R_t are electrical resistances at 0°C and $t^\circ\text{C}$.

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

where R_1 and R_2 are electrical resistances at temperatures t_1 and t_2 .

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^\circ\text{C}$$

where R_{100} is the resistance at 100°C .

Platinum resistance thermometer can measure temperature from -200°C to 1200°C .

(iii) Length of Mercury Column in a Capillary Tube

$$l_t = l_0(1 + \alpha t)$$

where α = coefficient of linear expansion and l_0 , l_t are lengths of mercury column at 0°C and $t^\circ\text{C}$.

Thermo Electro Motive Force

When two junctions of a thermocouple are kept at different temperatures, then a thermo-emf is produced between the junctions, which changes with temperature difference between the junctions. Thermo-emf

$$E = at + bt^2$$

where a and b are constants for the pair of metals.

Unknown temperature of hot junction when cold junction is at 0°C .

$$t = \left(\frac{E_t}{E_{100}} \times 100 \right)^\circ\text{C}$$

Where E_{100} is the thermo-emf when hot junction is at 100°C .

A thermo-couple thermometer can measure temperature from -200°C to 1600°C .

Thermal Equilibrium

When there is no transfer of heat between two bodies in contact, the the bodies are called in thermal equilibrium.

Zeroth Law of Thermodynamics

If two bodies A and B are separately in thermal equilibrium with a third body C, then bodies A and B will be in thermal equilibrium with each other.

Triple Point of Water

The values of pressure and temperature at which water coexists in equilibrium in all three states of matter, i.e., ice, water and vapour called triple point of water.

Triple point of water is 273 K temperature and 0.46 cm of mercury pressure.

Specific Heat

The amount of heat required to raise the temperature of unit mass of the substance through 1°C is called its specific heat.

It is denoted by c or s .

Its SI unit is joule/kilogram- $^\circ\text{C}$ ($\text{J}/\text{kg}\cdot^\circ\text{C}$). Its dimensions are $[\text{L}^2\text{T}^{-2}\theta^{-1}]$.

The specific heat of water is $4200 \text{ J kg}^{-1}\text{ }^\circ\text{C}^{-1}$ or $1 \text{ cal g}^{-1} \text{ }^\circ\text{C}^{-1}$, which is high compared with most other substances.

Gases have two types of specific heat

1. The specific heat capacity at constant volume (C_V).
2. The specific heat capacity at constant pressure (C_P).

Specific heat at constant pressure (C_P) is greater than specific heat at constant volume (C_V), i.e., $C_P > C_V$.

For molar specific heats $C_P - C_V = R$
where R = gas constant and this relation is called Mayer's formula.

The ratio of two principal specific heats of a gas is represented by γ .

$$\gamma = \frac{C_P}{C_V}$$

The value of γ depends on the atomicity of the gas.

Amount of heat energy required to change the temperature of any substance is given by

$$Q = mc\Delta t$$

- where, m = mass of the substance,
- c = specific heat of the substance and
- Δt = change in temperature.

Thermal (Heat) Capacity

Heat capacity of any body is equal to the amount of heat energy required to increase its temperature through 1°C .

Heat capacity = mc

where c = specific heat of the substance of the body and m = mass of the body.

Its SI unit is joule/kelvin (J/K).

Water Equivalent

It is the quantity of water whose thermal capacity is same as the heat capacity of the body. It is denoted by W .

$W = ms$ = heat capacity of the body.

Latent Heat

The heat energy absorbed or released at constant temperature per unit mass for change of state is called latent heat.

Heat energy absorbed or released during change of state is given by

$$Q = mL$$

where m = mass of the substance and L = latent heat.

Its unit is cal/g or J/kg and its dimension is $[\text{L}^2\text{T}^{-2}]$.

For water at its normal boiling point or condensation temperature (100°C), the latent heat of vaporisation is

$$\begin{aligned} L &= 540 \text{ cal/g} \\ &= 40.8 \text{ kJ/mol} \\ &= 2260 \text{ kJ/kg} \end{aligned}$$

For water at its normal freezing temperature or melting point (0°C), the latent heat of fusion is

$$\begin{aligned} L &= 80 \text{ cal/g} = 60 \text{ kJ/mol} \\ &= 336 \text{ kJ/kg} \end{aligned}$$

It is more painful to get burnt by steam rather than by boiling as 100°C gets converted to water at 100°C , then it gives out 536 heat. So, it is clear that steam at 100°C has more heat than water at 100°C (i.e., boiling of water).

After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt. So, in the mountains, when snow falls, one does not feel too hot but when ice melts, he feels too cold.

There is more shivering effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts, it absorbs large amount of heat from teeth.

Melting

Conversion of solid into liquid state at constant temperature is melting.

Evaporation

Conversion of liquid into vapour at all temperatures (even below boiling point) is called evaporation.

Boiling

When a liquid is heated gradually, at a particular temperature saturated vapour pressure of the liquid becomes equal to atmospheric pressure, now bubbles of vapour rise to the surface of the liquid. This process is called boiling of the liquid.

The temperature at which a liquid boils, is called boiling point. The boiling point of water increases with increase in pressure and decreases with decrease in pressure.

Sublimation

The conversion of a solid into vapour state is called sublimation.

Hoar Frost

The conversion of vapours into solid state is called hoar frost.

Calorimetry

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilocalories.

Principle of Calorimetry

When a hot body is mixed with a cold body, then heat lost by the hot body is equal to the heat gained by the cold body.

Heat lost = Heat gain

Thermal Expansion

Increase in size on heating is called thermal expansion. There are three types of thermal expansion.

1. Expansion of solids
2. Expansion of liquids
3. Expansion of gases

Expansion of Solids

Three types of expansion -takes place in solid.

Linear Expansion Expansion in length on heating is called linear expansion.

Increase in length

$$l_2 = l_1(1 + \alpha \Delta t)$$

where, l_1 and l_2 are initial and final lengths, Δt = change in temperature and α = coefficient of linear expansion.

Coefficient of linear expansion

$$\alpha = (\Delta l/l * \Delta t)$$

where l = real length and Δl = change in length and

Δt = change in temperature.

Superficial Expansion Expansion in area on heating is called superficial expansion.

Increase in area $A_2 = A_1(1 + \beta \Delta t)$

where, A_1 and A_2 are initial and final areas and β is a coefficient of superficial expansion.

Coefficient of superficial expansion

$$\beta = (\Delta A/A * \Delta t)$$

where. A = area, ΔA = change in area and Δt = change in temperature.

Cubical Expansion Expansion in volume on heating is called cubical expansion.

Increase in volume $V_2 = V_1(1 + \gamma \Delta t)$

where V_1 and V_2 are initial and final volumes and γ is a coefficient of cubical expansion.

Coefficient of cubical expansion

$$\gamma = \frac{\Delta V}{V \times \Delta t}$$

where V = real volume, ΔV = change in volume and Δt = change in temperature.

Relation between coefficients of linear, superficial and cubical expansions

$$\beta = 2\alpha \text{ and } \gamma = 3\alpha$$

$$\text{Or } \alpha:\beta:\gamma = 1:2:3$$

2. Expansion of Liquids

In liquids only expansion in volume takes place on heating.

(i) Apparent Expansion of Liquids When expansion of the container containing liquid, on heating is not taken into account then observed expansion is called apparent expansion of liquids.

Coefficient of apparent expansion of a liquid

$$(\gamma_a) = \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

(ii) Real Expansion of Liquids When expansion of the container, containing liquid, on heating is also taken into account, then observed expansion is called real expansion of liquids.

Coefficient of real expansion of a liquid

$$(\gamma_r) = \frac{\text{real increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

Both, γ_r , and γ_a are measured in $^{\circ}\text{C}^{-1}$.

We can show that $\gamma_r = \gamma_a + \gamma_g$

where, γ_r , and γ_a are coefficient of real and apparent expansion of liquids and γ_g is coefficient of cubical expansion of the container.

Anomalous Expansion of Water

When temperature of water is increased from 0°C , then its vol decreases upto 4°C , becomes minimum at 4°C and then increases. behaviour of water around 4°C is called, anomalous expansion water.

3. Expansion of Gases

There are two types of coefficient of expansion in gases

(i) Volume Coefficient (γ_v) At constant pressure, the change in volume per unit volume per degree celsius is called volume coefficient.

$$\gamma_V = \frac{V_2 - V_1}{V_0 (t_2 - t_1)}$$

where V_0 , V_1 , and V_2 are volumes of the gas at 0°C , $t_1^\circ\text{C}$ and $t_2^\circ\text{C}$.

(ii) Pressure Coefficient (γ_p) At constant volume, the change in pressure per unit pressure per degree celsius is called pressure coefficient.

$$\gamma_P = \frac{p_2 - p_1}{p_0 (t_2 - t_1)}$$

where p_0 , p_1 and p_2 are pressure of the gas at 0°C , $t_1^\circ\text{C}$ and $t_2^\circ\text{C}$.

Practical Applications of Expansion

1. When rails are laid down on the ground, space is left between the end of two rails.
2. The transmission cables are not tightly fixed to the poles.
3. The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel.
4. A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottles.

Important Points

- Due to increment in its time period a pendulum clock becomes slow in summer and will lose time.
- Loss of time in a time period $\Delta T = (1/2)\alpha \Delta\theta T$
 \therefore Loss of time in any given time interval t can be given by
 $\Delta T = (1/2)\alpha \Delta\theta t$
- At some higher temperature a scale will expand and scale reading will be lesser than true values, so that
 true value = scale reading $(1 + \alpha \Delta t)$
 Here, Δt is the temperature difference.
- However, at lower temperature scale reading will be more or true value will be less.

The branch of physics which deals with the study of transformation of heat energy into other forms of energy and vice-versa.

A thermodynamical system is said to be in thermal equilibrium when macroscopic variables (like pressure, volume, temperature, mass, composition etc) that characterise the system do not change with time.

Thermodynamical System

An assembly of an extremely large number of particles whose state can be expressed in terms of pressure, volume and temperature, is called thermodynamic system.

Thermodynamic system is classified into the following three systems

- (i) **Open System** It exchange both energy and matter with surrounding.
- (ii) **Closed System** It exchanges only energy (not matter) with surroundings.
- (iii) **Isolated System** It exchanges neither energy nor matter with the surrounding.

A thermodynamic system is not always in equilibrium. For example, a gas allowed to expand freely against vacuum. Similarly, a mixture of petrol vapour and air, when ignited by a spark is not an equilibrium state. Equilibrium is acquired eventually with time.

Thermodynamic Parameters or Coordinates or Variables

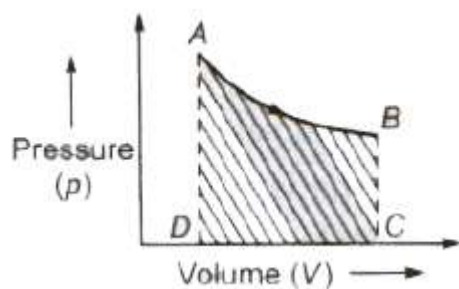
The state of thermodynamic system can be described by specifying pressure, volume, temperature, internal energy and number of moles, etc. These are called thermodynamic parameters or coordinates or variables.

Work done by a thermodynamic system is given by

$$W = p * \Delta V$$

where p = pressure and ΔV = change in volume.

Work done by a thermodynamic system is equal to the area enclosed between the p - V curve and the volume axis



Work done in process A-B = area ABCDA

Work done by a thermodynamic system depends not only upon the initial and final states of the system but also depend upon the path followed in the process.

Work done by the Thermodynamic System is taken as

Positive \rightarrow Δ as volume increases.

Negative \rightarrow Δ as volume decreases.

Internal Energy (U)

The total energy possessed by any system due to molecular motion and molecular configuration, is called its internal energy.

Internal energy of a thermodynamic system depends on temperature. It is the characteristic property of the state of the system.

Zeroth Law of Thermodynamics

According to this law, two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other. Thus, if A and B are separately in equilibrium with C, that is if $T_A = T_C$ and $T_B = T_C$, then this implies that $T_A = T_B$ i.e., the systems A and B are also in thermal equilibrium.

First Law of Thermodynamics

Heat given to a thermodynamic system (ΔQ) is partially utilized in doing work (ΔW) against the surrounding and the remaining part increases the internal energy (ΔU) of the system.

Therefore, $\Delta Q = \Delta U + \Delta W$

First law of thermodynamics is a restatement of the principle conservation of energy.

In isothermal process, change in internal energy is zero ($\Delta U = 0$).

Therefore, $\Delta Q = \Delta W$

In adiabatic process, no exchange of heat takes place, i.e., $\Delta\theta = 0$.

Therefore, $\Delta U = -\Delta W$

In adiabatic process, if gas expands, its internal energy and hence, temperature decreases and vice-versa.

In isochoric process, work done is zero, i.e., $\Delta W = 0$, therefore

$$\Delta Q = \Delta U$$

Thermodynamic Processes

A thermodynamical process is said to take place when some changes occur in the state of a thermodynamic system i.e., the thermodynamic parameters of the system change with time.

(i) Isothermal Process A process taking place in a thermodynamic system at constant temperature is called an isothermal process.

Isothermal processes are very slow processes.

These process follows Boyle's law, according to which

$$pV = \text{constant}$$

From $dU = nC_v dT$ as $dT = 0$ so $dU = 0$, i.e., internal energy is constant.

From first law of thermodynamic $dQ = dW$, i.e., heat given to the system is equal to the work done by system surroundings.

$$\text{Work done } W = 2.3026\mu RT \log_{10}(V_f / V_i) = 2.3026\mu RT \log_{10}(p_i / p_f)$$

where, μ = number of moles, R = ideal gas constant, T = absolute temperature and V_i , V_f and P_i , P_f are initial volumes and pressures.

After differentiating $P V = \text{constant}$, we have

$$\frac{dp}{dV} = -\frac{p}{V} \text{ and } -\frac{dp}{dV} = \frac{p}{V}$$

i.e., bulk modulus of gas in isothermal process, $\beta = p$.

$P - V$ curve for this process is a rectangular hyperbola

Examples

(a) Melting process is an isothermal change, because temperature of a substance remains constant during melting.

(b) Boiling process is also an isothermal operation.

(ii) **Adiabatic Process** A process taking place in a thermodynamic system for which there is no exchange of heat between the system and its surroundings.

Adiabatic processes are very fast processes.

These process follows Poisson's law, according to which

$$pV^\gamma = TV^{\gamma-1} = \frac{T^\gamma}{p^{\gamma-1}} = \text{constant}$$

From $dQ = nC_dT$, $C_{\text{adi}} = 0$ as $dQ = 0$, i.e., molar heat capacity for adiabatic process is zero.

From first law, $dU = -dW$, i.e., work done by the system is equal to decrease in internal energy. When a system expands adiabatically, work done is positive and hence internal energy decrease, i.e., the system cools down and vice-versa.

Work done in an adiabatic process is

$$W = \frac{nR(T_i - T_f)}{\gamma - 1} = \frac{p_i V_i - p_f V_f}{\gamma - 1}$$

where T_i and T_f are initial and final temperatures. Examples

- (a) Sudden compression or expansion of a gas in a container with perfectly non-conducting wall.
- (b) Sudden bursting of the tube of a bicycle tyre.
- (c) Propagation of sound waves in air and other gases.

(iii) **Isobaric Process** A process taking place in a thermodynamic system at constant pressure is called an isobaric process.

Molar heat capacity of the process is C_p and $dQ = nC_p dT$.

Internal energy $dU = nC_v dT$

From the first law of thermodynamics

$$dQ = dU + dW$$

$$dW = pdV = nRdT$$

Process equation is $V / T = \text{constant}$.

p- V curve is a straight line parallel to volume axis.

(iv) **Isochoric Process** A process taking place in a thermodynamic system at constant volume is called an isochoric process.

$dQ = nC_v dT$, molar heat capacity for isochoric process is C_v .

Volume is constant, so $dW = 0$,

Process equation is $p / T = \text{constant}$

p - V curve is a straight line parallel to pressure axis.

(v) **Cyclic Process** When a thermodynamic system returns to its initial state after passing through several states, then it is called cyclic process.

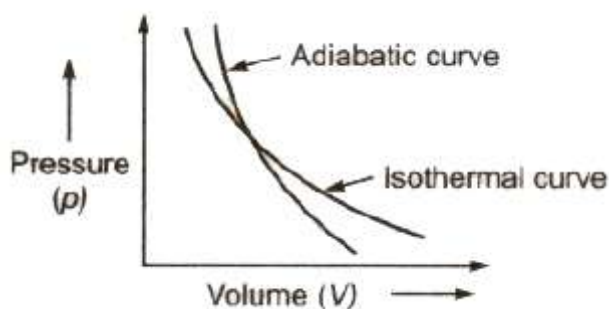
Efficiency of the cycle is given by

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

Work done by the cycle can be computed from area enclosed cycle on p - V curve.

Isothermal and Adiabatic Curves

The graph drawn between the pressure p and the volume V of a given mass of a gas for an isothermal process is called **isothermal curve** and for an adiabatic process it is called **adiabatic curve**.



The slope of the adiabatic curve

$= \gamma \times$ the slope of the isothermal curve

Volume Elasticities of Gases

There are two types of volume elasticities of gases

(i) Isothermal modulus of elasticity $E_s = p$

(ii) Adiabatic modulus of elasticity $E_T = \gamma p$

Ratio between isothermal and adiabatic modulus

$$E_S / E_T = \gamma = C_p / C_v$$

where C_p and C_v are specific heats of gas at constant pressure and at constant volume.

For an isothermal process $\Delta t = 0$, therefore specific heat,

$$c = \Delta \theta / m \Delta t = \infty;$$

For an adiabatic process $\Delta \theta = 0$, therefore specific heat,

$$c = 0 / m \Delta t = 0$$

Second Law of Thermodynamics

The second law of thermodynamics gives a fundamental limitation to the efficiency of a heat engine and the coefficient of performance of a refrigerator. It says that efficiency of a heat engine can never be unity (or 100%). This implies that heat released to the cold reservoir can never be made zero.

Kelvin's Statement

It is impossible to obtain a continuous supply of work from a body by cooling it to a temperature below the coldest of its surroundings.

Clausius' Statement

It is impossible to transfer heat from a lower temperature body to a higher temperature body without use of an external agency.

Planck's Statement

It is impossible to construct a heat engine that will convert heat completely into work.

All these statements are equivalent as one can be obtained from the other.

Entropy

Entropy is a physical quantity that remains constant during a reversible adiabatic change.

Change in entropy is given by $dS = \delta Q / T$

Where, δQ = heat supplied to the system

and T = absolute temperature.

Entropy of a system never decreases, i.e., $dS \geq 0$.

Entropy of a system increases in an irreversible process

Heat Engine

A heat energy engine is a device which converts heat energy into mechanical energy.

A heat engine consists of three parts

- (i) Source of heat at higher temperature
- (ii) Working substance
- (iii) Sink of heat at lower temperature

Thermal efficiency of a heat engine is given by

$$\eta = \frac{\text{Work done / cycle}}{\text{Total amount of heat absorbed / cycle}}$$
$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where Q_1 is heat absorbed from the source,

Q_2 is heat rejected to the sink and T_1 and T_2 are temperatures of source and sink.

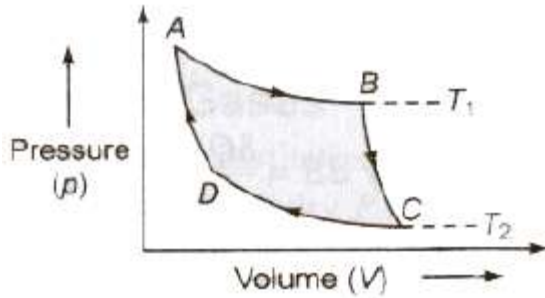
Heat engine are of two types

(i) **External Combustion Engine** In this engine fuel is burnt a chamber outside the main body of the engine. e.g., steam engine. In practical life thermal efficiency of a steam engine varies from 12% to 16%.

(ii) **Internal Combustion Engine** In this engine. fuel is burnt inside the main body of the engine. e.g., petrol and diesel engine. In practical life thermal efficiency of a petrol engine is 26% and a diesel engine is 40%.

Carnot's Cycle

Carnot devised an ideal cycle of operation for a heat engine, called Carnot's cycle.



A Carnot's cycle contains the following four processes

- (i) Isothermal expansion (AB)
- (ii) Adiabatic expansion (BC)
- (iii) Isothermal compression (CD)
- (iv) Adiabatic compression (DA)

The net work done per cycle by the engine is numerically equal to the area of the loop representing the Carnot's cycle .

After doing the calculations for different processes we can show that

$$\frac{\theta_2}{\theta_1} = \frac{T_2}{T_1}$$

Therefore, efficiency of the cycle is

$$\eta = 1 - \frac{T_2}{T_1}$$

[Efficiency of Carnot engine is maximum (not 1000/0) for given temperatures T_1 and T_2 . But still Carnot engine is not a practical engine because many ideal situations have been assumed while designing this engine which can practically not be obtained.]

Refrigerator or Heat Pump

A refrigerator or heat pump is a device used for cooling things. It absorb heat from sink at lower temperature and reject a larger amount of heat to source at higher temperature.

Coefficient of performance of refrigerator is given by

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

where Q_2 is heat absorbed from the sink, Q_1 is heat rejected to source and T_1 and T_2 are temperatures of source and sink.

Relation between efficiency (η) and coefficient of performance (β)

$$\beta = \frac{1 - \eta}{\eta}$$

ELECTROSTATICS : Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics

- **STATIC CLING**
- An electrical phenomenon that accompanies dry weather, causes these pieces of papers to stick to one another and to the plastic comb.
- Due to this reason our clothes stick to our body.
- **ELECTRIC CHARGE** : Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes.
- Charge cannot exist without material carrying it
- It is possible to develop the charge by **rubbing two solids having friction**.
- Carrying the charges is called **electrification**.
- Electrification due to friction is called **frictional electricity**.

Since these charges are not flowing it is also called static electricity.

There are two types of charges. +ve and -ve.

- Similar charges repel each other,
- Opposite charges attract each other.
- Benjamin Franklin made this nomenclature of charges being +ve and -ve for mathematical calculations because adding them together cancel each other.
- Any particle has vast amount of charges.
- The number of positive and negative charges are **equal**, hence **matter is basically neutral**.
- Inequality of charges give the material a **net** charge which is equal to the difference of the two type of charges.

Electrostatic series :If two substances are rubbed together the former in series acquires the positive charge and later, the -ve.

(i) Glass (ii) Flannel (iii) Wool (iv) Silk (v) Hard Metal (vi) Hard rubber (vii) Sealing wax (viii) Resin (ix) Sulphur

Electron theory of Electrification

- Nucleus of atom is positively charged.
- The electron revolving around it is negatively charged.
- They are equal in numbers, hence atom is electrically neutral.
- With friction there is transfer of electrons, hence net charge is developed in the particles.
- It also explains that the charges are compulsorily developed in pairs equally. +ve in one body and -ve in second.
- It establish **conservation of charges in the universe**.
- The **loss** of **electrons** develops +ve charge. While **excess** of **electrons** develop -ve charge
- A **proton** is 1837 times heavier than electron hence it cannot be transferred. Transferring lighter electron is easier.
- Therefore for electrification of matter, only **electrons** are active and responsible.

Charge and Mass relation

- Charge cannot exist without matter.
- One carrier of charge is electron which has **mass** as well.
- Hence if there is charge transfer, mass is also transferred.
- Logically, negatively charged body is heavier than positively charged body.

Conductors, Insulators and Semiconductors

- **Conductors** : Material in which electrons can move easily and freely.

Ex. Metals, Tap water, human body.

Brass rod in our hand, if charged by rubbing the charge will move easily to earth. Hence Brass is a conductor.

The flow of this excess charge is called **discharging**

- **Insulator** : Material in which charge cannot move freely. Ex . Glass, pure water, plastic etc.

- Electrons can be forced to move across an insulator by applying strong force (called electric field.) Then this acts like a conductor.

- dielectric strength.**

The maximum electric field an insulator can withstand without becoming a conductor is called its dielectric strength.

- Semiconductor** : is a material which under little stimulation (heat or Elect. Field) converts from insulator to a conductor.

Ex. Silicon, germanium.

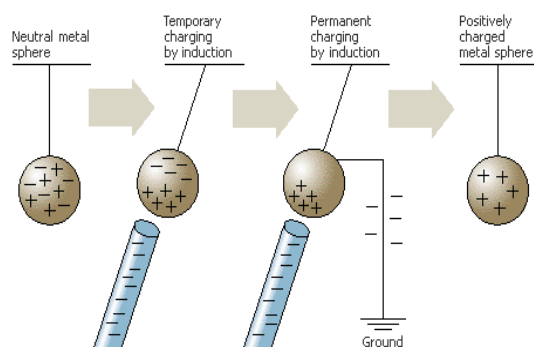
- Superconductor** : is that material which presents no resistance to the movement of the charge through it.

The resistance is precisely zero.

Electrostatic Induction

- Phenomenon of polarization of charges in a body, when a charged body is present near it, is called electrostatic induction.
- In this process bodies are charged without touching them.

Charging by Induction



A charged object will induce a charge on a nearby conductor. In this example, a negatively charged rod pushes some of the negatively charged electrons to the far side of a nearby copper sphere because like charges repel each other. The positive charges that remain on the near side of the sphere are attracted to the rod.

- If the sphere is grounded so that the electrons can escape altogether, the charge on the sphere will remain if the rod is removed.

Basic properties of Electric charge

- Additivity of Electric charges
- Quantization of Electric charge
- Conservation of Electric Charge

Additivity of Charges..

- Charges can be added by simple rules of algebra. Addition of positive and negative charge makes Zero charge

Quantization of Electric charge

- Principle: **Electric charge is not a continuous quantity, but is an integral multiple of minimum charge (e).**
- Reason of quantization:
- Minimum charge e exist on an electron.
- The material which is transferred during electrification is an electron, in integral numbers.
- Hence **charge transferred has to be integral multiple of e .**
- Charge on an electron ($-e$) and charge on a proton ($+e$) are equal and opposite, and are the **minimum**.

This minimum charge is 1.6×10^{-19} coulomb.

one electron has charge -1.6×10^{-19} C

One proton has charge $+1.6 \times 10^{-19}$ C

- Charge on a body Q is given by

$$Q = \pm ne$$

Where n is a whole number 1,2,3.....

and $e = 1.6 \times 10^{-19}$

- since e is smallest value of charge, it is called Elementary Charge or Fundamental charge

- (**Quarks** : In new theories of proton and neutrons, a required constituent particles called Quarks which carry charges $\pm(1/3)e$ or $\pm(2/3)e$.

- But because free quarks do not exist and their sum is always an integral number, it does not violate the quantization rules.)

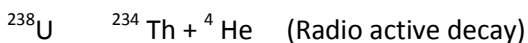
Conservation of Charges

- Like conservation of energy, and Momentum, the electric charges also follow the rules of conservation.

1. Isolated (Individual) Electric charge can neither be created nor destroyed, it can only be transferred.
2. Charges in pair can be created or destroyed.

Example for 1.

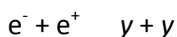
At Nuclear level : Decay of U-238



Atomic number Z of radioactive material U-238 is 92. Hence it has 92 protons hence charge is 92e. Thorium has Z= 90, hence charge is 90e, alpha particles have charge 2e. Therefore charges before decay are 92 and after decay are 90+2=92

Example for 2. (a) Annihilation (destruction in pair)

In a nuclear process an electron -e and its antiparticle positron +e undergo annihilation process in which they transform into two gamma rays (high energy light)



Example for 2 (b):Pair production:

is converse of annihilation, charge is also conserved when a gamma ray transforms into an electron and a positron



Electric Force - Coulomb's Law

- Coulomb's law in Electrostatics :

Force of Interaction between two stationary point charges is

directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

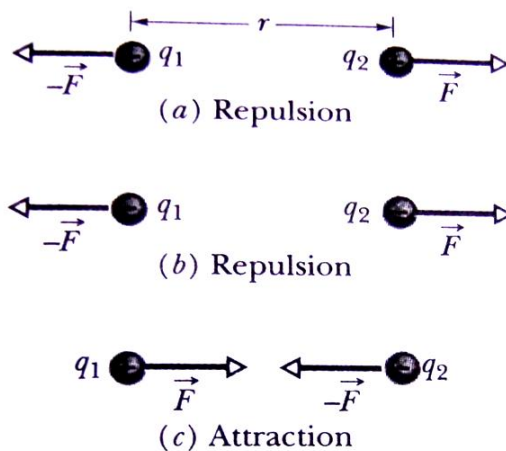


Fig. 22-6 Two charged particles, separated by distance r , repel each other if their charges are (a) both positive and (b) both negative. (c) They attract each other if their charges are of opposite signs. In each of the three situations, the force acting on one particle is equal in magnitude to the force acting on the other particle but has the opposite direction.

If two charges q_1 and q_2 are placed at distance r then,

$$F = c \frac{q_1 q_2}{r^2}$$

where c is a constant .

c is called Coulomb's constant and its value is

$$c = \frac{1}{4\pi\epsilon_0} \quad \therefore \quad F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

The value of c depends upon system of units and on the medium between two charges

It is seen experimentally that if two charges of 1 Coulomb each are placed at a distance of 1 meter in air or vacuum, then they attract each other with a force (F) of 9×10^9 Newton.

Accordingly value of c is 9×10^9 Newton x m^2/coul^2

ϵ_0 is permittivity of free space or vacuum and its value is $\epsilon_0 = 8.85 \times 10^{-12} \text{ coul}^2 / \text{N x m}^2$

If point charges are immersed in a dielectric medium, then ϵ_0 is replaced by ϵ a quantity-characteristic of the matter involved In such case. For vacuum $\epsilon = \epsilon_0$

$$F = \frac{1}{4\pi\epsilon} \frac{q_1q_2}{r^2}$$

Permittivity, Relative Permittivity and Dielectric Constant

Permittivity is a measure of the property of the medium surrounding electric charge which determine the forces between the charges.

Its value is known as Absolute permittivity of that Medium ϵ

More is Permittivity of medium, Less is coulombs Force.

For water, permittivity is 80 times then that of vacuum, hence force between two charges in water will be 1/80 time force in vacuum (or air.)

Relative Permittivity(ϵ_r) : It is a dimension-less characteristic constant, which express absolute permittivity of a medium w.r.t. permittivity of vacuum or air. It is also called

Dielectric constant (K) $K = \epsilon_r = \epsilon/\epsilon_0$

- This result leads to the calculation that

$$\epsilon_r = K = \frac{F \text{ (Force in Vacuum)}}{F' \text{ (Force in Medium)}}$$

$$F = \frac{1}{4\pi\epsilon} \frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0K} \frac{q_1q_2}{r^2}$$

- Unit of charge:- In S.I. System of units, the unit of charge is **Coulomb**.
- **One coulomb** is defined as that charge, which, when placed at a distance of 1 m in air or vacuum from an equal and similar charge, repel it with a force of 9×10^9 Newton
- Charge on one electron is 1.6019×10^{-19} coul. Hence
- One coulomb is equivalent to a charge of 6.243×10^{18} electrons

Is electric charge a fundamental quantity?

- No, In S.I. System, the fundamental quantity is **Electric current** and its unit is Ampere. Therefore coulomb is defined in it's terms as under:
- Coulomb is that quantity of charge which passes across any section of a conductor per second when current of one ampere flows through it, i.e.
- 1 coulomb = 1 Ampere x 1 sec

In **cgs electrostatic** system, the unit of charge is called as STATECOULUMB or esu of charge.

- In this system electrostatic constant $c=1$ for

$$F = \frac{q_1q_2}{r^2}$$

vacuum or air,

One stat coulomb is defined that amount of charge which when placed at a distance of 1 cm in air from an equal and similar charge repel it with a force of one dyne.

In **cgs electromagnetic** system, the unit of charge is called ABCOULOMB or emu of charge

$$1 \text{ Coulomb} = 3 \times 10^9 \text{ statcoulomb} = 1/10 \text{ abcoulomb}$$

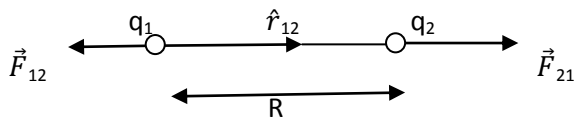
$$\backslash 1 \text{ emu} = 3 \times 10^{10} \text{ esu of charge}$$

Vector form of Coulombs' Law

Equation of Coulombs force showing magnitude as well as direction is called Vector form of coulombs' law.

If \hat{r}_{12} is unit vector pointing from q_1 to q_2 , then as per diagram \hat{r}_{12} and \vec{F}_{21} will be in the same direction, then

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2} \hat{r}_{12} \text{ (vector equation) } \dots\dots\dots 1.$$



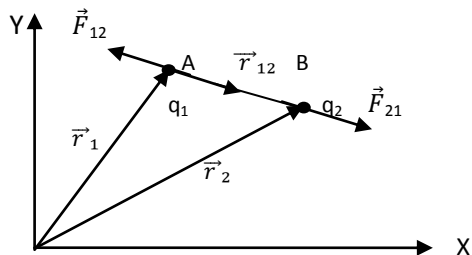
$$\text{Similarly } \vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2} \hat{r}_{21} \dots\dots\dots 2$$

$$\text{Since } \hat{r}_{21} = - \hat{r}_{12} \quad \therefore \vec{F}_{21} = - \vec{F}_{12}$$

Electrostatic Force between two point charges in terms of their position vectors.

(i). Let there be two point charges q_1 and q_2 at points A & B in vacume. With reference to an origin O let their

position vectors be \vec{r}_1 (OA) and \vec{r}_2 (OB). Then $AB = \vec{r}_{12}$. According to triangle law of vectors :
 $\vec{r}_1 + \vec{r}_{12} = \vec{r}_2 \quad \therefore \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ and
 $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$



(ii) According to Coulomb's law, the Force \vec{F}_{12} exerted on q_1 by q_2 is given by : $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$ where \hat{r}_{21} is a unit vector pointing from q_2 to q_1 . We know that $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$

Hence, general Vector forms of Coulomb's equation is

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{|\vec{r}_1 - \vec{r}_2|^2} (\vec{r}_1 - \vec{r}_2) \quad \text{and}$$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{|\vec{r}_2 - \vec{r}_1|^2} (\vec{r}_2 - \vec{r}_1)$$

Comparison of Electrostatic and Gravitational Force

1. Identical Properties :

- Both the forces are central forces, i.e., they act along the line joining the centers of two charged bodies.
- Both the forces obey inverse square law, $F \propto \frac{1}{r^2}$
- Both are conservative forces, i.e. the work done by them is independent of the path followed.
- Both the forces are effective even in free space.

2. Non identical properties :

- a. Gravitational forces are always attractive in nature while electrostatic forces may be attractive or repulsive.
- b. Gravitational constant of proportionality does not depend upon medium, the electrical constant of proportionality depends upon medium.
- c. Electrostatic forces are extremely large as compared to gravitational forces

Qn. Compare electrostatic and gravitational force between one electron and one proton system.

Ans : $F_e = \frac{1}{4\pi\epsilon_0} \frac{e.e}{r^2} = 9 \times 10^9 \frac{(1.6 \times 10^{-19})^2}{r^2}$ Newton

$F_g = G \frac{m_e \times m_p}{r^2} = 6.67 \times 10^{-11} \frac{(9.1 \times 10^{-31}) \times (1.67 \times 10^{-27})}{r^2}$ Newton

$\frac{F_e}{F_g} = 2.26 \times 10^{39}$

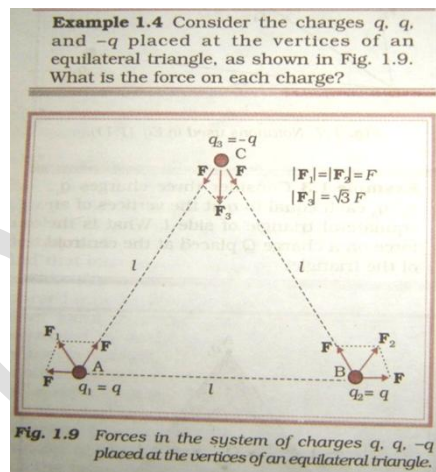
Principle of Superposition of Charges :

If a number of Forces $F_{11}, F_{12}, F_{13}, \dots, F_{1n}$ are acting on a single charge q_1 then charge will experience force F_1 equal to vector sum of all these forces .

$F_1 = F_{11} + F_{12} + F_{13} + \dots + F_{1n}$

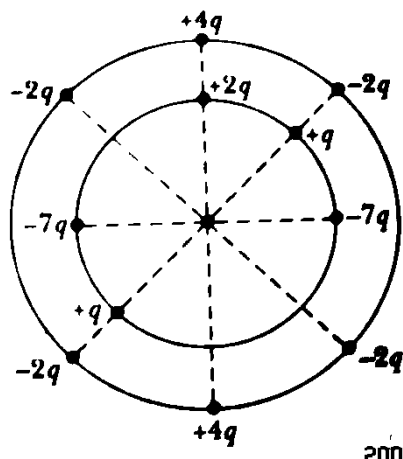
The vector sum is obtained as usual by parallelogram law of vectors.

All electrostatics is basically about Coulomb's Law and Principle of superposition.



NUMERICALS FOR PRACTICE

1. How many electrons must be removed from the sphere to give it a charge of $+2 \mu C$. Is there any change in the mass when it is given this positive charge. How much is this change?
2. Two identical charged copper spheres A and B have their centers separated by a distance of 50 cm. A third sphere of same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B?
3. A central particle of charge $-q$ is surrounded by two circular rings of charged particles, of radii r and R , such that $R > r$. What are the magnitude and direction of the net electrostatic force on the central particle due to other particles.



4.-Three equal charges each of 2.0×10^{-6} are fixed at three corners of an equilateral triangle of side 5 cm. Find the coulomb force experienced by one of the charges due to other two.

5.



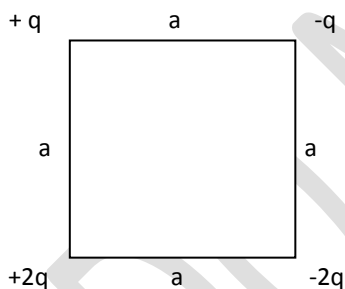
Above two charged particles are free to move. At one point, however a third charged particle can be placed such that all three particles are in equilibrium.

- (a) is that point to the left of the first two particles, to their right, or between them?
- (b) Should the third particle be positively or negatively charged?
- © Is the equilibrium stable or unstable?

6. A charge q is placed at the center of the line joining two equal charges Q . Show that the system of three charges will be in equilibrium if $q = Q/4$.

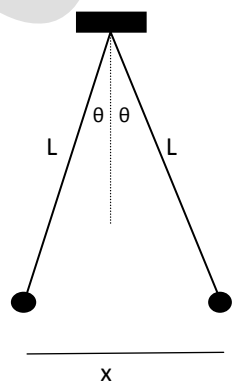
7. Two particles having charges $8q$ and $-2q$ are fixed at a distance L . where, in the line joining the two charges, a proton be placed so that it is in equilibrium (the net force is zero). Is that equilibrium stable or unstable?

8. What are the horizontal and vertical components of the net electrostatic force on the charged particle in the lower left corner of the square if $q = 1.0 \times 10^{-7}C$ and $a = 5.0$ cm?



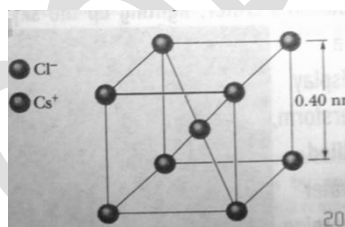
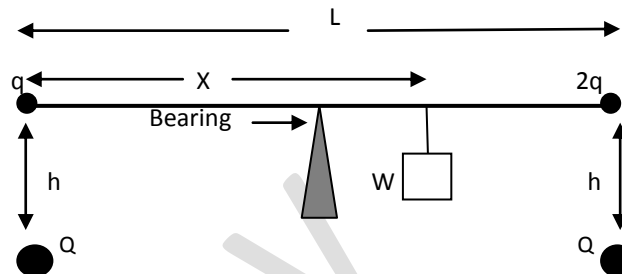
9. Two tiny conducting balls of identical mass m and identical charge q hang from non conducting threads of length L . Assume that θ is so small that $\tan \theta$ can be replaced by $\sin \theta$; show that, for equilibrium,

$$x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$$



8.A long non-conducting massless rod of length L , pivoted at its centre and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive

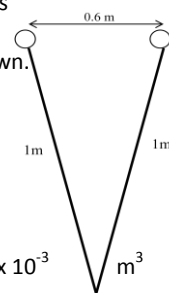
charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q . a. Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?



9. In the basic CsCl (Cesium chloride) crystal, Cs^+ ions form the corners of a cube and a Cl^- ion is at the centre of cube. Edge length is 0.40 nm.

- (a) What is the magnitude of the net electrostatic force exerted on Cl^- ion by the eight Cs^+ ions?
- (b) If one of the Cs^+ ion is missing the crystal is said to have defect. How much will be the force on chlorine ion in that case?

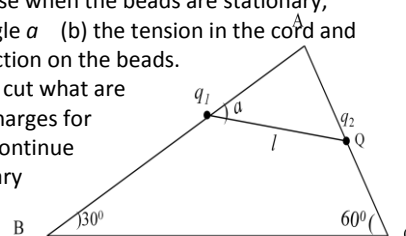
10. Two similar helium-filled spherical balloons tied to a 5 g weight with strings and each carrying a charge q float in equilibrium as shown. Find (a) the magnitude of q , assuming that the charge on each balloon is at its centre and (b) the volume of each balloon. Assume that the density of air $= 1.29 \text{ kg m}^{-3}$ and the density of helium in the balloon is $= 0.2 \text{ kg m}^{-3}$. Neglect the weight of the unfilled balloons. Ans: $q = 5.5 \times 10^{-7}$ $V = 2.3 \times 10^{-3}$



11. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density of 800 kg m^{-3} , the angle remain the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1600 kg m^{-3} Ans : $K = 2$

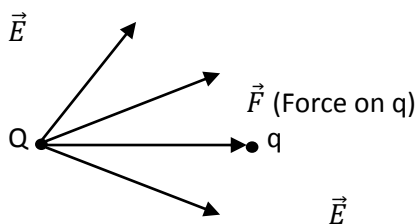
12. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane. Two beads of equal masses m each and carrying charges q_1 q_2 are connected by a cord of length l and can slide without friction on the wires. Considering the case when the beads are stationary, determine (a) angle α (b) the tension in the cord and (c) the normal reaction on the beads.

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary



ELECTRIC FIELD

ELECTRIC FIELD is the environment created by an electric charge (source charge) in the space around it, such that if any other electric charges (test charges) are present in this space, it will come to know of its presence and exert a force on it.



INTENSITY (OR STRENGTH) OF ELECTRIC FIELD AT A LOCATION is the force exerted on a unit charge placed at that location

: if intensity of electric field at a location is E and a charge 'q' is placed, then force experienced by this charge F is

$$\vec{F} = q \cdot \vec{E} \quad \text{--- 1}$$

or

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{--- 2}$$

Direction of force F is in direction of electric field E

$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{r} \quad \text{--- 3}$$

By equ.1 and 3 : Intensity of electric field due to Source charge Q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r} \quad \text{--- 4}$$

By Coulomb's law we know that in similar situation if q=1 then

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Relation in F, E and Test charge q is $E = \frac{F}{q}$

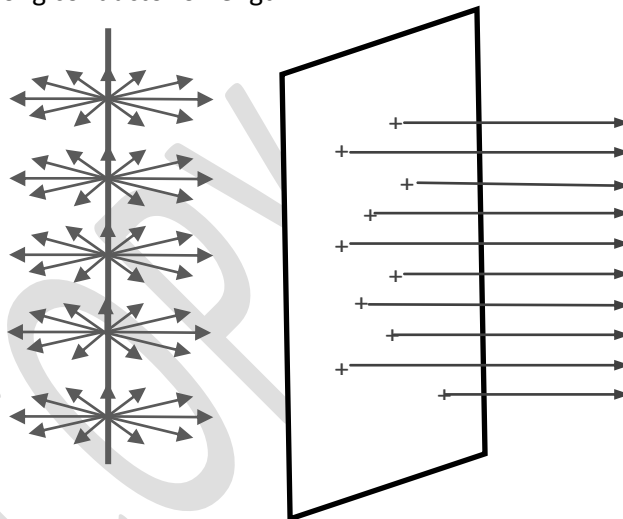
DISTRIBUTION OF CHARGE

Electric charge on a body may be concentrated at a point, then it is called a 'point charge'. If it is distributed all over, then it is called distribution of

charge. Depending on shape of it is given different names

1. Linear distribution: when charge is evenly distributed over a length. In such case we use a quantity Linear charge density λ . Which has relation

$\lambda = \frac{Q}{L}$, Where 'Q' is charge distributed over a long conductor of length 'L'



2- Areal distribution: charge is evenly distributed over a surface area, S.

The surface charge density is 'σ' given by $\sigma = \frac{Q}{S}$

Where Q is charge given to a surface of area 'S'.

3- volumetric distribution: charge is evenly distributed throughout the body having volume 'V' Volumetric charge density is 'ρ' $\rho = \frac{Q}{V}$

GENERAL DISTRIBUTION OF ELECTRIC FIELD DUE TO DIFFERENT DISTRIBUTION OF CHARGES

1-Due to point charge Q $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$

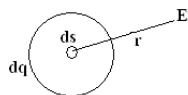
2-E due to linear distribution of electric charge

$$dq = \lambda \cdot dl$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda \cdot dl}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \int_L \frac{\lambda \cdot dl}{r^2}$$

3 - E due to areal distribution of charge:

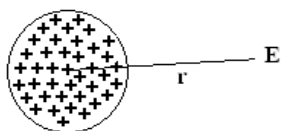


$$dq = \sigma \cdot ds$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\sigma \cdot ds}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \int_s \frac{\sigma \cdot ds}{r^2}$$

4- E due to volumetric distribution of charge



SOLID SPHERE VOLUME V

$$E = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho \cdot ds}{r^2}$$

DIPOLE

1-Dipole is a system of two equal and opposite charges at finite & fixed distance.

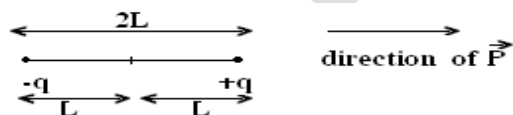
example: molecule of electrolytic compounds.
Example - HCl, H₂O.

2-CO₂ & CH₄ are non-polar because centers of -ve & +ve charges co-incide and there is no distance between them.

3-if non polar atom is placed in an elect.field a distance is created between +ve & -ve charge: it become polar.

Dipole moment:-the effectiveness or strength of a dipole is measured by the physical quantity .Dipole moment \vec{P} . it is calculated as $\vec{P} = q \times 2\vec{L}$

$P = q \times 2L$ (magnitude) or $\vec{P} = q \times 2\vec{L}$ (vector)

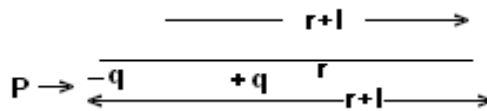


Where 'q' is each charge and '2L' is distance between them.(each charge is at a distance L from 'center' of dipole)

Dipole moment $\vec{P} = q \times 2\vec{L}$ is a vector quantity it has magnitude $p=2qL$

And its direction is along line from -q to +q.

ON THE AXIAL LINE



E DUE TO +q
ALONG \vec{P}

$$E_1 = \frac{q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$$

E DUE TO -q

$$E_2 = \frac{-q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$$

OPPOSITE TO \vec{P}
NET ELECTRIC FIELD

$$E = E_1 + E_2 = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4rl}{(r^2 - l^2)^2} \quad \{ 2ql = P \}$$

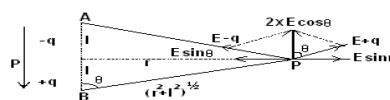
$$E = \frac{2P \cdot r}{4\pi\epsilon_0 (r^2 - l^2)^2}$$

SINCE $E_1 > E_2$

\vec{E} IS IN THE DIRECTION OF \vec{P}

IF $R \gg L$ THE, $E = \frac{2P}{4\pi\epsilon_0 r^3}$

2 \vec{E} ON EQUATORIAL LINE (TRANSVERAL LINE)



E due to +q, $E_{+q} = \frac{q}{4\pi\epsilon_0 (r^2 - l^2)} \hat{BP}$

E due to -q, $E_{-q} = \frac{q}{4\pi\epsilon_0 (r^2 - l^2)} \hat{PA}$

$|E_{+q}| = |E_{-q}| = E_q$

each E_q is resolved in two direction. One along equatorial line and other in axial directions which are the $E \sin\theta$ and normal direction $E \cos\theta$.

ELECTRIC FIELD DUE TO DIPOLE

$E \sin \theta$ in opposite direction cancel each other while
 $E \cos \theta$ add up to two.

: net electric field $E = 2E \cos \theta$

$$E(\text{net}) = 2E \cos \theta$$

$$= 2 \cdot \frac{q}{4\pi\epsilon_0(r+l)^2} \frac{l}{(r+l)^2}^{1/2}$$

$$E = \frac{P}{4\pi\epsilon_0 (r^2+l^2)^{3/2}}$$

IF $R \gg L$ Then,
$$E = \frac{P}{4\pi\epsilon_0 r^3}$$

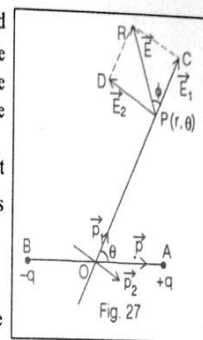
The direction is opposite to that of P

Electric Field at equatorial line is half of the field on axial line in strength and opposite in direction.

z

Electric Field Intensity due to a Short Electric Dipole at some General Point

- (i) Let AB be a short electric dipole of dipole moment \vec{p} (directed from B to A). We are interested to find the electric field at some general point P whose polar coordinates are (r, θ) . The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ .
- (ii) We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components \vec{p}_1 and \vec{p}_2 as shown in Fig. 27, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$.
 The magnitudes of \vec{p}_1 and \vec{p}_2 are $p_1 = p \cos \theta$ and $p_2 = p \sin \theta$.
- (iii) It is clear from figure that point P lies on the axial line of dipole with moment \vec{p}_1 . Hence magnitude of the electric field intensity \vec{E}_1 at P due to \vec{p}_1 is



$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p \cos \theta}{r^3} \quad (\text{along } \vec{p}_1) \quad \dots\dots(1)$$

Similarly, P lies on the equatorial line of dipole with moment \vec{p}_2 . Hence, magnitude of electric field intensity \vec{E}_2 at P due to \vec{p}_2 is

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \sin \theta}{r^3} \quad (\text{opposite to } \vec{p}_2) \quad \dots\dots(2)$$

Hence resultant intensity at P is : $\vec{E} = \vec{E}_1 + \vec{E}_2$

Magnitude of \vec{E} is : $E = \sqrt{(E_1^2 + E_2^2)}$ (as \vec{E}_1 and \vec{E}_2 are mutually perpendicular).

$$\text{or } E = \sqrt{\left(\frac{2p \cos \theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p \sin \theta}{4\pi\epsilon_0 r^3}\right)^2} = \frac{p}{4\pi\epsilon_0 r^3} \sqrt{4 \cos^2 \theta + \sin^2 \theta}$$

$$\text{or } E = \frac{p}{4\pi\epsilon_0 r^3} \sqrt{1 + 3 \cos^2 \theta} \quad \dots\dots(3)$$

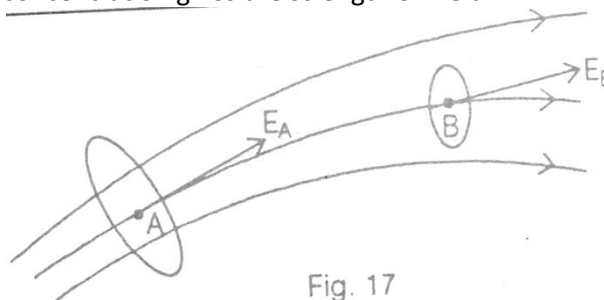
- (iv) If the resultant field intensity vector \vec{E} makes an angle ϕ with the direction of \vec{E}_1 , then

$$\tan \phi = \frac{E_2}{E_1} = \frac{(p \sin \theta / 4\pi\epsilon_0 r^3)}{(2p \cos \theta / 4\pi\epsilon_0 r^3)} = \frac{1}{2} \tan \theta$$

Electric Line of Force :

The idea of Lines of Force was given by Michel Faraday. These are imaginary lines which give visual idea of Electric field, its magnitude, and its direction.

A line of force is continuous curve the tangent to which at a point gives the direction of Electric field, and its concentration gives the strength of Field.

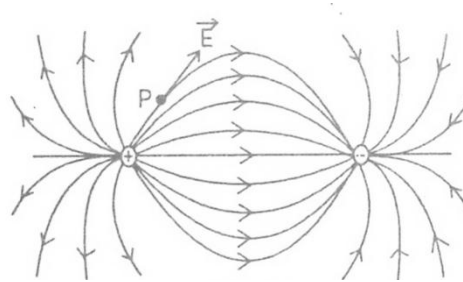


Electric Field at A is stronger than field at B.

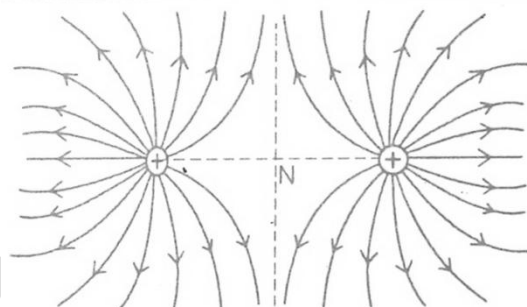
Properties of Electric Lines of Force :

Electric Lines of Force :

- 1.start from positive charge and end at negative.
- 2.Electric Lines of forces are **imaginary** but Electric field they represent is **real**.
- 3.The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point.
- 4.**In SI system, the number of electric lines originating or terminating on charge q is q/ϵ_0 . That means lines associated with unit charge are $1/\epsilon_0$**
- 5.Two lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.
6. Electric Lines of force can never be a closed loop since they do not start and end at the same point. The lines are discontinuous, start from + and terminate at -
7. The electric line of force do not pass through a conductor as electric field inside a conductor is zero.
8. Lines of force have tendency to contract longitudinally like a stretched string, producing attraction between opposite charges and edge effect.
- 9.Electric Lines of force start and end **Normal to the surface** of conductor.
10. Crowded lines represent strong field while distant lines represent weak field. Equidistant parallel lines represent uniform field. Non-straight or non- parallel represent non-uniform field. In the diagram a is uniform while b, c, and d are non-uniform fields.



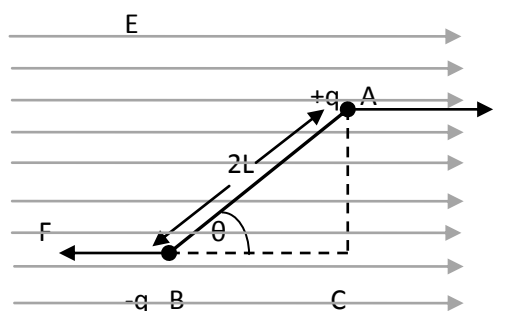
Lines of force due to Two positive charges



Elect field lines due to straight line distribution :
And Electric field lines due to very large sheet of charge are shown in the previous page.

Electric dipole in electric field

When a dipole is placed in an electric field each charge experience a force ($F=qe$) . Positive, in the direction of field and negative, opposite to direction of field.

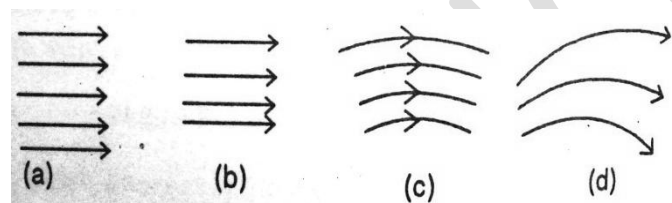


Net Force on dipole : $F + (-F) = 0$ zero

Hence dipole will not make any linear motion.

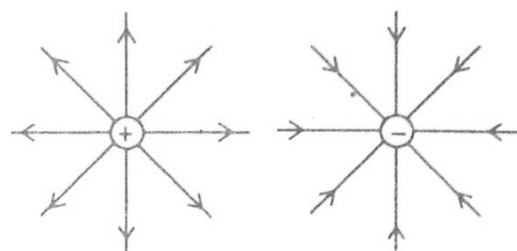
Torque on dipole: A couple of force is acting on the body of dipole system at different points, the forces are equal and opposite in uniform field. Hence they form a couple of forces which create a **torque**. Therefore dipole is capable of rotation in a uniform electric field. The moment of forces or Torque is

$$\tau = F \times AC = qEx2L\sin\theta = 2qL E \sin\theta = PE\sin\theta$$



Field Lines due to some charge configurations.

- 1.Single positive or negative charge



Two equal and opposite charges :

or $\vec{\tau} = \mathbf{P} \times \mathbf{E}$

NOTE :

1. Direction of torque is normal to the plane containing dipole moment P and electric field E and is governed by right hand screw rule.

2. If Dipole is parallel to E the torque is **Zero**.

3. Torque is **maximum** when Dipole is perpendicular to E and that torque is PE

4. This equation gives the definition of dipole moment. If E is 1 N/C then $P=T$.

Therefore; **Dipole Moment of a dipole is equal to the Torque experience by that dipole when placed in an electric field of strength 1 N/C at right angle to it.**

5. If a dipole experiencing a torque in electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reach along the direction of E the torque become zero. But due to inertia it overshoots this equilibrium condition and then starts oscillating about this mean position.

6. *Dipole in Non-Uniform Electric field :*

In case Electric field is non-uniform, magnitude of force on $+q$ and $-q$ will be different, hence a net force will be acting on centre of mass of dipole and it will make a linear motion. At the same time due to couple of forces acting, a torque will also be acting on it.

Work done in rotating a dipole in a uniform Electric field:

1. If a dipole is placed in a uniform electric field experience a torque. If it is rotated from its equilibrium position, work has to be done on it. If an Electric dipole with moment P is placed in electric field E making an angle α , then torque acting on it at that instant is

$$\tau = PE \sin \alpha$$

2. If it is rotated further by a small angle $d\alpha$ then work done $dw = (PE \sin \alpha) \cdot d\alpha$

Then work done for rotating it through an angle θ from equilibrium position of angle 0 is :-

$$W = \int_0^\theta (PE \sin \alpha) \cdot d\alpha = PE [-\cos \alpha]_0^\theta$$

$$\text{Or, } W = PE [-\cos \theta + \cos 0] = pE [1 - \cos \theta]$$

3. If a dipole is **rotated through 90°** from the direction of the field, then work done will be

$$W = pE [1 - \cos 90] = pE$$

4. If the dipole is **rotated through 180°** from the direction of the field, then work done will be :

$$W = pE [1 - \cos 180] = 2 pE$$

Potential Energy of a dipole kept in Electric field :

1. dipole in Equilibrium (P along E):-

A dipole is kept in Electric field in equilibrium condition, dipole moment P is along E

To calculate Potential Energy of dipole we calculate work done in bringing $+q$ from zero potential i.e. ∞ to location B , and add to the work done in bringing $-q$ from ∞ to position A .

1. The work done on $-q$ from ∞ up to A

$$= -(Work \text{ done up to } B + Work \text{ done from } B \text{ to } A)$$

2. Work done on $+q = +(Work \text{ done up to } B)$

Adding the two

$$\begin{aligned} \text{Total work done} &= \text{Work done on } -q \text{ from } B \text{ to } A \\ &= \text{Force} \times \text{displacement} \\ &= -qE \times 2L = -2qLE \\ &= -P \cdot E \end{aligned}$$

This work done convert into Potential Energy of dipole

$$U = -\vec{P} \cdot \vec{E}$$

If P and E are inclined at angle θ to each other then magnitude of this Potential Energy is

$$U = -P E \cos \theta$$

Electric – Potential

- (1) Electric Potential is characteristic of a location in the electric field. If a unit charge is placed at that location it has potential energy (due to work done on its placement at that location). This potential energy or work done on unit charge in bringing it from infinity is called potential at that point.

(2) Potential – Difference (i) is the work done on unit charge for carrying it from one location to other location A.



$$V_A \longleftarrow \text{-----} q V_\infty$$

Potential at A ----- V_A

Energy with q at A is $q V_A$

Energy with Q at B is $q V_B$

Difference of Energy $U_A - U_B = q (V_A - V_B)$

Using work energy theorem . $W = q ((V_A - V_B))$

Or, $V_A - V_B = W / q$ & $U_A - U_B = W$.

If $V_B = 0$ { At ∞ Potential $V = 0$, Inside Earth $V_E = 0$ }

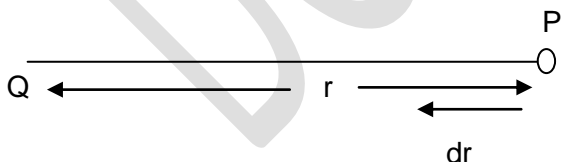
Then $V_A = W / q$

This equation gives definition of potential V at point A as under :-

“Potential of a point in electric field is the work done in bringing a unit charge from infinity (Zero potential) to that point, without any acceleration.”

Expression of potential at a point due to source charge Q :-

Let there be a charge Q which creates electric field around it. Point P is at distance ‘ r ’ from it. Let’s calculate potential at this point.



A test charge ‘ q ’ is moved for a small displacement dr towards Q .

Electric field due to Q at P , $E = \frac{Q}{4\pi\epsilon_0 r^2}$

To move it against this electrical force we have to apply force in opposite direction

Hence applied force $F = - \frac{Qq}{4\pi\epsilon_0 r^2}$

Work done in moving distance dr is

$$dw = - \frac{Qq}{4\pi\epsilon_0 r^2} dr$$

Total work done in bringing the charge from distance ∞ to distance r is

$$W = - \int_\infty^r \frac{Qq}{4\pi\epsilon_0 r^2} dr$$

$$= - \frac{Qq}{4\pi\epsilon_0} \int_\infty^r \frac{dr}{r^2}$$

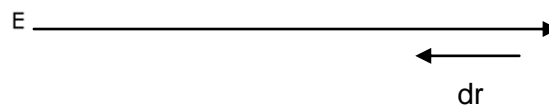
$$= - \frac{Qq}{4\pi\epsilon_0} \left[\frac{r^{-1}}{-1} \right]_\infty^r = \frac{Qq}{4\pi\epsilon_0 r}$$

$$W/q = \frac{Q}{4\pi\epsilon_0 r} \quad \text{OR} \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

Where Q is source charge, r is distance & V_r is potential at that point.

Basically V_r is also a “potential difference” between potential of this point P and Potential at ∞ (i.e., 0).

Relation between E & V



A test charge q is moved against E for a small distance dr . then work done dw by applied force $-qE$ is $dw = - qE dr$

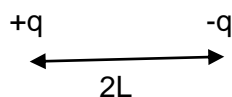
Or, $dw / q = - E dr$
 Or, $dV = - E.dr$
 Or, $E = - dv / dr$

Electric field is derivative of potential difference. –ve sign show that direction of E is opposite to direction of dv . I.e., dv decrease along the direction of E



$$V_A > V_B$$

This also show that an electric charge experience force from high potential towards low potential if allowed to move, it will do so in this direction only.



At P - $V_{+q} = \frac{Q}{4\pi \epsilon_0 (r-l)}$

$V_{-q} = \frac{Q}{4\pi \epsilon_0 (r+l)}$

Total $V = -V_{+q} + V_{-q} = \frac{Q}{4\pi \epsilon_0} \left(\frac{1}{r-l} - \frac{1}{r+l} \right)$
 $= \frac{2Ql}{4\pi \epsilon_0 (r^2 - l^2)} = \frac{P}{4\pi \epsilon_0 (r^2 - l^2)}$

If $r \gg L$ Then $V = \frac{P}{4\pi \epsilon_0 r^2}$

If E and are not collinear and make angle θ between them, then according to relation of work & force

$dv = - E dr \cos \theta$

Or, $- dv / dr = E \cos \theta$

Or, $dv = - E . dr$



Or $V = E . dr$

Or { Potential difference is a scalar quantity (work) given by dot product of two vector

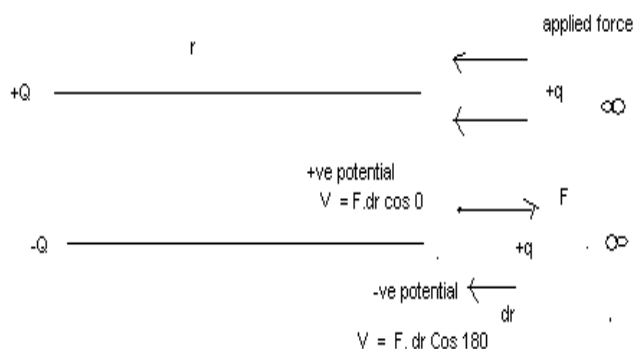
\vec{E} & \vec{dr} .

Principle of super position:-

1) Potential at a point due to different charges is Algebraic sum of potentials due to all individual charges.

$V = V_1 + V_2 + V_3$

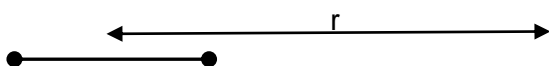
2) Potential due to +ve charge is +ve



Potential due to -ve charge is -ve

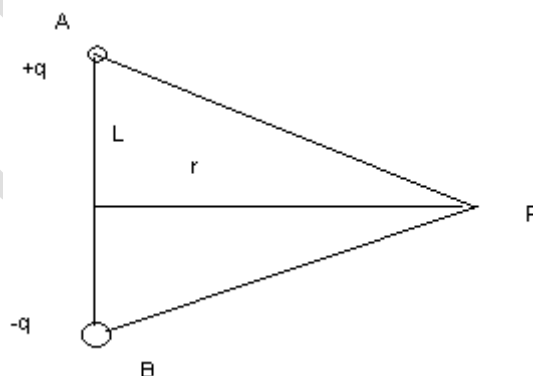
Potential due to a dipole

1) At a point on axial line:-



2) At a point on equatorial line

-q & +q are placed at A & B. Point P is on equatorial

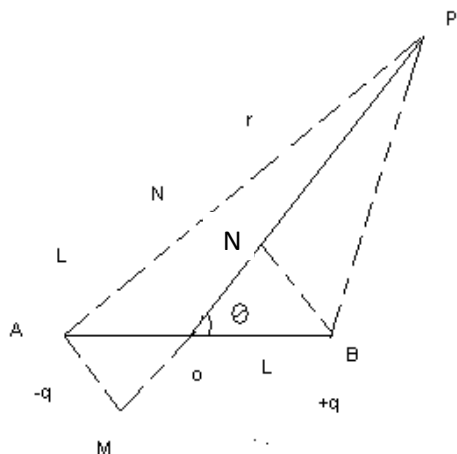


line

Every point on equatorial line is equidistant from +q & -q. Therefore +ve & -ve potential are equal **Hence net potential is zero.**

“Potential at every point on equatorial line of dipole is zero.”

iii) Potential due dipole at any general point.



Draw normal from A & B on PO

$$PB \approx PN = PO - ON = r - L \cos \theta \text{ ----- (i)}$$

$$PA \approx PM = PO + OM = r + L \cos \theta \text{ ----- (ii)}$$

$$V_{+q} = \frac{Q}{4\pi \epsilon_0 PB} = \frac{Q}{4\pi \epsilon_0 (r - L \cos \theta)}$$

$$V_{-q} = \frac{-Q}{4\pi \epsilon_0 PA} = \frac{-Q}{4\pi \epsilon_0 (r + L \cos \theta)}$$

Total $V = V_{+q} + V_{-q} =$

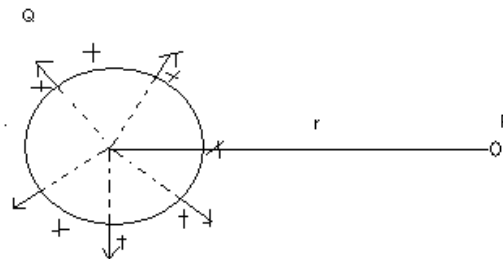
$$\begin{aligned} & \frac{Q}{4\pi \epsilon_0} \left(\frac{1}{r - L \cos \theta} - \frac{1}{r + L \cos \theta} \right) \\ &= \frac{Q}{4\pi \epsilon_0} \left(\frac{r + L \cos \theta - r + L \cos \theta}{r^2 - L^2 \cos^2 \theta} \right) \\ &= \frac{Q \times 2L \cos \theta}{4\pi \epsilon_0 (r^2 - L^2 \cos^2 \theta)} \end{aligned}$$

Or $V = \frac{P \cos \theta}{4\pi \epsilon_0 (r^2 - L^2 \cos^2 \theta)}$

If $r \gg L$

Then, Or, $V = \frac{P \cos \theta}{4\pi \epsilon_0 r^2}$

Potential due to spherical shell



A spherical shell is given charge Q. The electric field is directed normal to surface i.e., Radially outward. "Hence charge on the surface of a shell behaves as if all the charge is concentrated at centre.

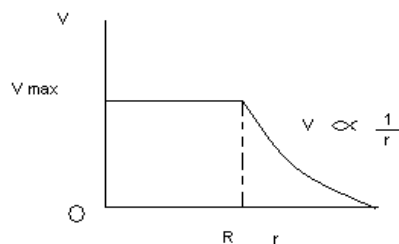
Hence potential at distance r is $V = \frac{Q}{4\pi \epsilon_0 r}$

Potential on the surface of shell $V = \frac{Q}{4\pi \epsilon_0 R}$

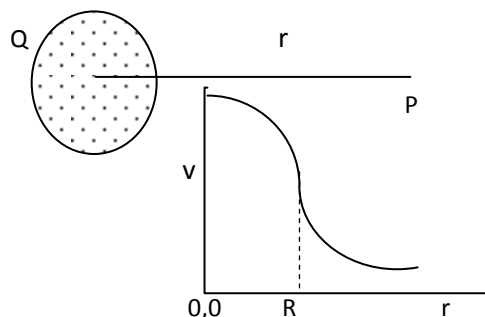
Inside shell Electric field is Zero. Therefore change in potential $dv = \text{Zero} \times dr = 0$ i.e., No change in potential. Hence potential inside a spherical shell is same as on the surface and it is same at every point.

It is $V = \frac{Q}{4\pi \epsilon_0 R}$ Where R is radius of shell.

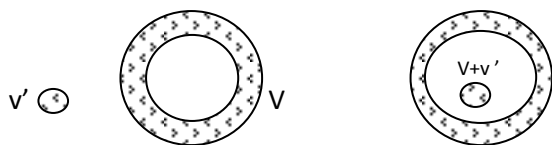
Relation of V & r for spherical shell



In case of non-conducting sphere of charge. potential keeps on increasing up to centre as per diagram.



A body of potential v' is placed inside cavity of shell with potential V then potential of the body become $V+v'$



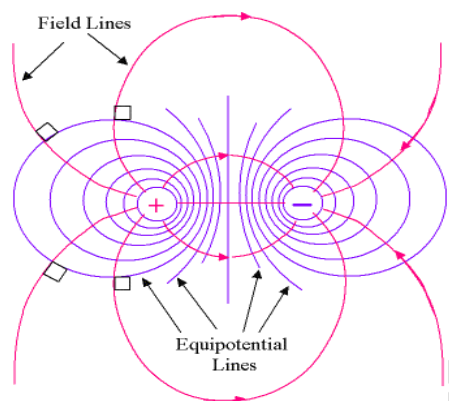
.Equipotential Surface

A real or imaginary surface in an electric field which has same potential at very point is an equipotential surface or simply, an equipotential.

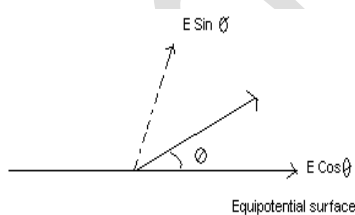
Ex:- A shell having electric charge at its centre, makes an equipotential surface as it has same potential

$$\frac{Q}{4\pi \epsilon_0 R}$$

at every point of the surface.



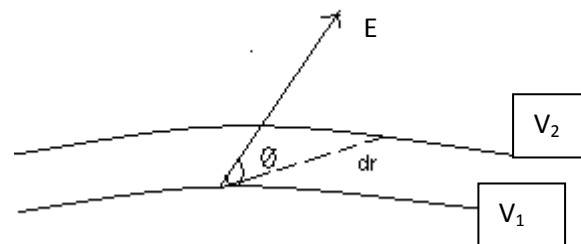
Electric lines of force and equipotential surface are at right angle to each other.



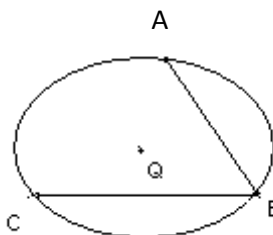
Proof:- Suppose E is not at right angle to equipotential surface, and makes angle θ with it. Then it has two components, $E \cos \theta$ along surface and $E \sin \theta$ normal to surface due to component $E \cos \theta$, force $q E \cos \theta$ should be created on surface and it should move the charge. But we find that charges are in equilibrium. i.e.

$E \cos \theta = 0$;
 since $E \neq 0$, therefore $\cos \theta = 0$ or $\angle \theta = 90^\circ$
 Hence E is always at right angle to equip. surface.

ii) $V_2 - V_1 = dv = - E \cos \theta \cdot dr$



iii) No work is done in carrying an electric charge from one point of E.P. Surface to other point (Whatever is the path)



Net work done in carrying charge from A to B is Zero, B to C is Zero, because $W = qV$ and V is same on this equipotential Surface

iv) Surface of a conductor in electrostatic field is always an equipotential surface.

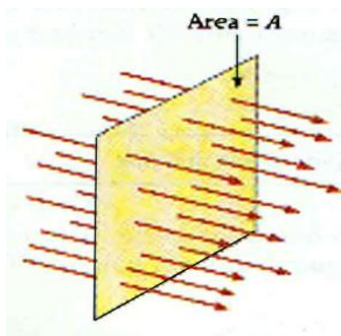
Distribution of charge on uneven surface: - charge density is more on the surface which is pointed, or has smaller radius. Therefore if a conductor is brought near pointed charged surface, due to high density of charge induction will be more. Electric field set up will be very strong. This leads to construction of use of lightning arrester used on the buildings.

Gauss's Law
Electric Flux

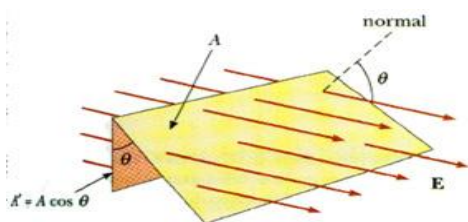
Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux**".

We will define the **electric flux** Φ for an electric field that is perpendicular to an area as

$\Phi = E A$



If the electric field **E** is **not** perpendicular to the area, we will have to modify this to account for that.



Think about the "air flux" of air passing through a window **at an angle** θ . The "effective area" is $A \cos \theta$ or the component of the velocity perpendicular to the window is $v \cos \theta$. With this in mind, we will make a general definition of the electric flux as

$$\Phi = E A \cos \theta$$

Φ You can also think of the electric flux as the **number** of electric field lines that cross the surface.

Remembering the "dot product" or the "scalar product", we can also write this as

$$\Phi = \vec{E} \cdot \vec{A}$$

where **E** is the electric field and **A** is a vector equal to the area **A** and in a direction **perpendicular** to that area. Sometimes this same information is given as

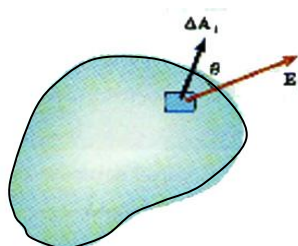
$$\vec{A} = A \vec{n}$$

where **n** is a **unit vector** pointing **perpendicular** to the area. In that case, we could also write the electric flux across an area as

$$\Phi = \vec{E} \cdot \vec{n} A$$

Both forms say the same thing. For this to make any sense, we must be talking about an area where the **direction** of **A** or **n** is constant.

For a curved surface, that will not be the case. For that case, we can apply this definition of the electric flux over a small area ΔA or ΔA or ΔA_n .



Then the electric flux through that small area is $\Delta \Phi$ and

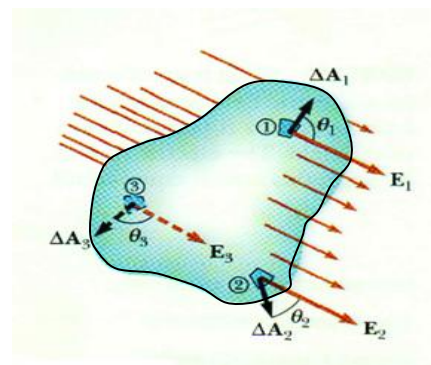
$$\Delta \Phi = E \Delta A \cos \theta \text{ or}$$

$$\Delta \Phi = \vec{E} \cdot \Delta \vec{A}$$

To find the flux through all of a closed surface, we need to sum up all these contributions of $\Delta \Phi$ over the entire surface,

$$\Phi_c = \oint \vec{E} \cdot d\vec{A} = \oint E_n dA \cos \theta$$

We will consider flux as **positive** if the electric field **E** goes from the inside to the outside of the surface and we will consider flux as **negative** if the electric field **E** goes from the outside to the inside of the surface. This is important for we will soon be interested in the **net** flux passing through a surface.



Gauss's Law : Total electric flux through a closed surface is $1/\epsilon_0$ times the charge enclosed in the surface.

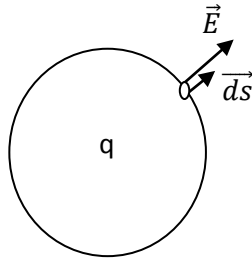
$$\Phi_E = q / \epsilon_0$$

But we know that Electrical flux through a closed surface is $\oint \vec{E} \cdot d\vec{s}$

$$\therefore \oint \vec{E} \cdot d\vec{s} = q / \epsilon_0$$

This is Gauss's theorem.

PROOF : Let's consider an hypothetical spherical surface having charge q placed at its centre. At every point of sphere the electrical field is radial, hence making angle 0 degree with area vector.



$$\text{At the small area flux } d\phi = \oint \vec{E} \cdot \vec{ds}$$

$$= \oint E \cdot ds \cdot \cos 0^\circ$$

$$= \oint \frac{q}{4\pi\epsilon_0 r^2} ds \quad (E = \frac{q}{4\pi\epsilon_0 r^2}, \cos 0 = 1)$$

$$= \frac{q}{4\pi\epsilon_0 r^2} \oint ds$$

For a sphere $\oint ds$ is $4\pi r^2$.

$$\therefore \Phi = \frac{q}{4\pi\epsilon_0 r^2} \times 4\pi r^2.$$

$$\text{Or, } \Phi = q / \epsilon_0$$

This is Gauss Theorem. (Hence proved)



Application of Gauss's Law

To calculate Electric Field due to different charge distributions.

For this purpose we consider construction of a **Gaussian surface**.

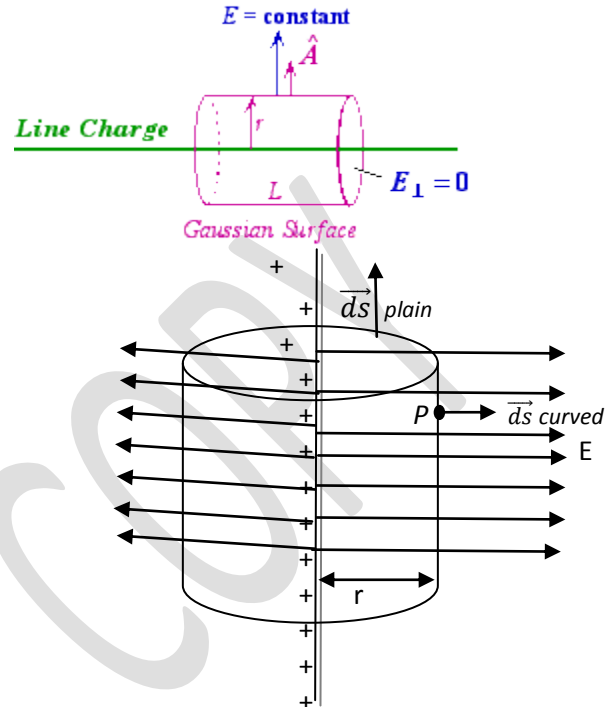
Gaussian Surface : It is an imaginary surface in the electric field which is

1. closed from all sides
2. Surface is Symmetrical about the charges in it
3. Electric field \vec{E} on the surface is symmetrical

Electric field due to line charge :

Electric charge is distributed on an infinite long straight conductor with linear charge density λ . We have to find Electric field on a point P at normal distance r .

Consider a Gaussian Surface in the shape of a cylinder having axis along conductor. It has radius r so that point P lies on the surface. Let its length be l . The electric field is normal to conductor, hence it is symmetrical to the surfaces of these cylinder.



Now $\oint \vec{E} \cdot \vec{ds} = \int \vec{E} \cdot \vec{ds}$ for curved surface + $\int \vec{E} \cdot \vec{ds}$ for 2 plane surfaces.

$$= \int E \cdot ds \cos 0 + \int E \cdot ds \cos 90$$

$$= E \int ds \text{ for curved surface (E is uniform)}$$

$$= E 2\pi r l \quad (\int ds = 2\pi r l, \text{ for cylindrical curved surface})$$

The charge enclosed within Gaussian surface $= \lambda l$

$$\text{According to Gauss theorem : } \oint \vec{E} \cdot \vec{ds} = q / \epsilon_0$$

$$\text{Putting values : } E 2\pi r l = \lambda l / \epsilon_0$$

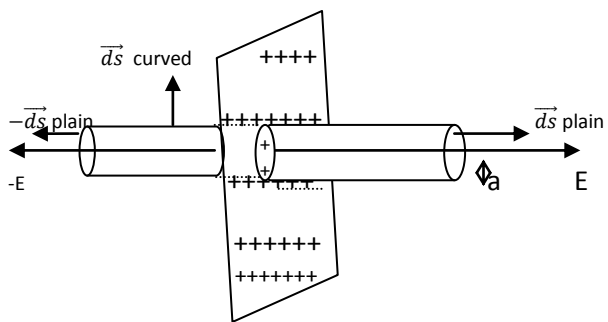
$$\text{Or, } \boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

Electric field due to a plain surface :-

There is a very large plain surface having surface density σ . There is a point P at normal distance r .

Let's consider a Gaussian surface, in shape of a cylinder which has axis normal to the sheet of charge and containing point P at its plain surface (radius a).

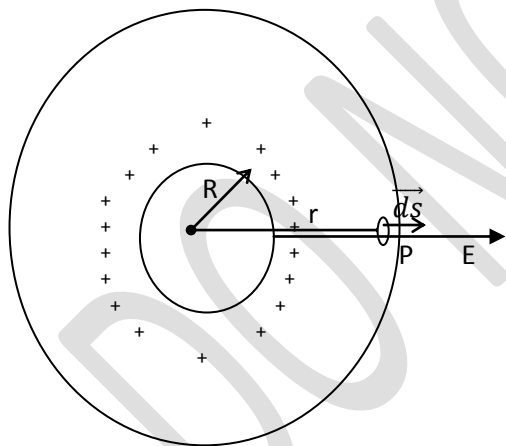
Electric field E is normal to the surface containing charge hence it is normal to the plain surface of cylinder and parallel to curved surface.



Now $\oint \vec{E} \cdot d\vec{s} = \int \vec{E} \cdot d\vec{s}$ for curved surface + $\int \vec{E} \cdot d\vec{s}$ for 2 plane surfaces.
 $= \int E \cdot ds \cos 90 + \int E \cdot ds \cos 0 + \int -E \cdot (-ds \cos 0)$
 = for plain surfaces $2E \int ds$ (E is uniform)
 $= 2E\pi a^2$
 The charge enclosed inside Gaussian surface $q = \sigma \cdot A$
 Or, $q = \sigma \pi a^2$

Applying Gauss's Law : $\oint \vec{E} \cdot d\vec{s} = q / \epsilon_0$
 Putting values $2E\pi a^2 = \frac{\sigma \pi a^2}{\epsilon_0}$
 Or $E = \frac{\sigma}{2\epsilon_0}$

Electric Field due to charge distributed over a spherical shell :-



The spherical shell or spherical conductor has total charge q, surface charge density σ , radius R. We have to find Electric Field E at a point P at distance 'r'.

Case 1. If P is outside shell.

Let's assume a Gaussian surface, which is a concentric sphere of radius r and P lies on its surface. Electric field is normal to surface carrying charge. Hence it is radially outward. Therefore for a small area on the Gaussian surface ds E is normal to surface i.e. angle between $d\vec{s}$ and \vec{E} is 0.

Now $\oint \vec{E} \cdot d\vec{s} = \int \vec{E} \cdot d\vec{s}$ for complete area of Gaussian surface
 $= \int E \cdot ds \cdot \cos 0 = E \int ds$ (E is uniform)
 $= E \times 4\pi r^2$. (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = q

Applying Gauss's Law : $\oint \vec{E} \cdot d\vec{s} = q / \epsilon_0$
 Putting values $E \times 4\pi r^2 = q / \epsilon_0$

Or $E = \frac{q}{4\pi\epsilon_0 r^2}$

This expression is same as electric field due to a point charge q placed at distance r from P. i.e. In this case if complete charge q is placed at the centre of shell the electric field is same.

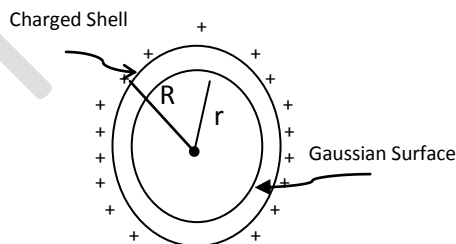
Case 2. If P is on the surface.

In above formula when r decrease to R the electric field increase.

On the surface (replace r with R) $E = \frac{q}{4\pi\epsilon_0 R^2}$

Hence this is electric field on the surface of a shell and its value is maximum compared to any other point.

Case 3. If P is within the surface. Or 'r' < R



Let's consider a Gaussian surface, a concentric spherical shell of radius r passing through P.

Then charge contained inside Gaussian surface is Zero.

According to Gauss's Theorem $\oint \vec{E} \cdot d\vec{s} = q / \epsilon_0$

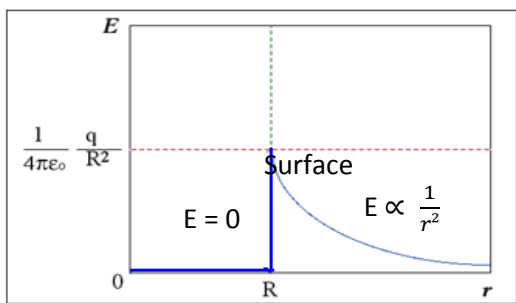
If q is zero then $\oint \vec{E} \cdot d\vec{s} = 0$.

As ds is not zero then $E = 0$

It is very important conclusion reached by Gauss's Law that Electric field inside a charged shell is zero.

The electric field inside conductor is Zero. This phenomenon is called **electrostatic shielding**.

Variation of E with r (distance from centre)



Putting values $E \times 4\pi r^2 = \frac{\rho}{3} \pi r^2 / \epsilon_0$

$$\therefore E = \frac{\rho r}{3\epsilon_0}$$

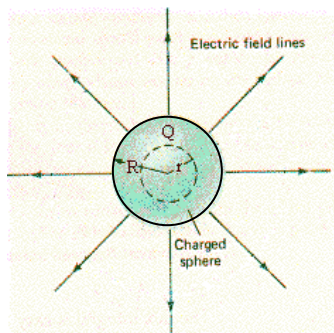
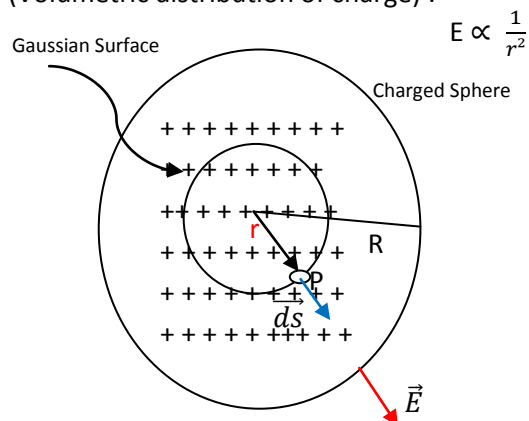
It shows that inside a sphere of charge, the electric field is directly proportional to distance from centre.

At centre $r=0 \therefore E=0$

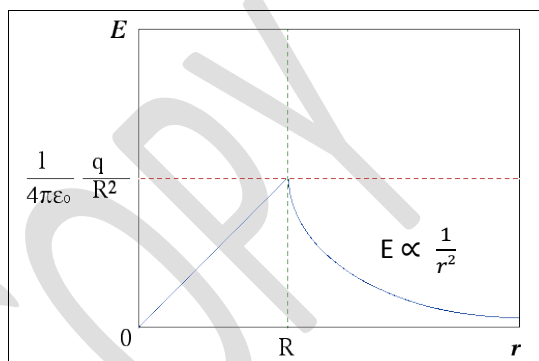
On the surface $E = \frac{\rho R}{3\epsilon_0} = \frac{q}{4\pi\epsilon_0 R^2}$ ($\rho = q / \frac{4}{3}\pi r^3$)

Electric Field due to (filled-up) sphere of charge

(Volumetric distribution of charge) :

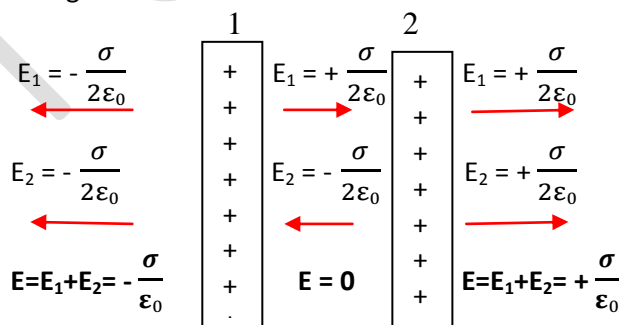


Variation of E with r (distance from centre)

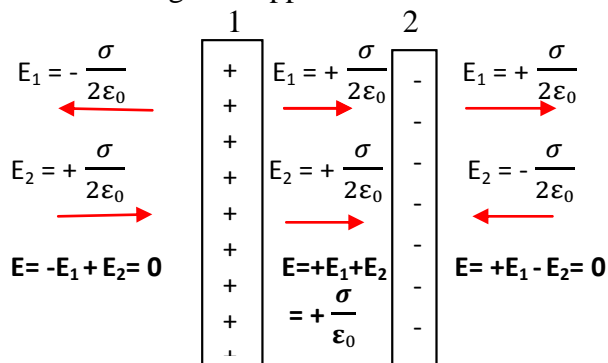


Electric field due to two charged parallel surface

Charges of similar nature



a. Charges of opposite nature :-



Case 1. When P is Out side sphere. Same as in the case of charged shell $E = \frac{q}{4\pi\epsilon_0 r^2}$

Case 2. When point P is on the surface of shell: Same as in case of shell . $E = \frac{q}{4\pi\epsilon_0 R^2}$

Case 3 If point P is inside the charged sphere.

Consider Gaussian surface, a concentric spherical shell of radius r, such that point P lies on the surface. Electric field is normal to the surface.

Now $\oint \vec{E} \cdot \vec{ds} = \int \vec{E} \cdot \vec{ds}$ for complete area of Gaussian surface
 $= \int E \cdot ds \cdot \cos 0 = E \int ds$ (E is uniform)
 $= E \times 4\pi r^2$. (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = charge density x volume.

$= \rho \frac{4}{3}\pi r^3$ (where ρ is the charge per unit volume.)

Applying Gauss's Law $\oint \vec{E} \cdot \vec{ds} = q / \epsilon_0$

Equipotential Surface :

Energy of a charged particle in terms of potential:-

Work required to bring a charge q at a point of potential V is $W = qV$. This work done on the charged particle converts to its potential energy.

Potential energy of charge q at potential V is $U = qV$

Electron-Volt : By relation Work/energy = qV , the smallest unit of work/energy is Electron Volt.

One electron volt is the work done by/on one electron for moving between two points having potential difference of one Volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$$

Potential Energy of system of charges

(i) System of Two charges :

A B
 q_1 ----- r ----- q_2
 Potential due to q_1 at B is potential at distance r :
 $V = \frac{q_1}{4\pi\epsilon_0 r}$ \therefore Potential Energy of system $U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$

(ii) System of three charges

We make different pairs and calculate energy as under

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}}$$

(iii) System of Four charges

Four charges make six pairs : Potential Energy $U =$

$$\frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}} + \frac{q_1 q_4}{4\pi\epsilon_0 r_{14}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}} + \frac{q_2 q_4}{4\pi\epsilon_0 r_{24}} + \frac{q_3 q_4}{4\pi\epsilon_0 r_{34}}$$

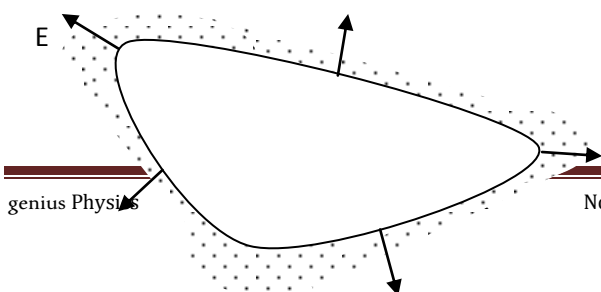
The energy is contained in the system and not by any one member. But it can be used by one or more members.

Distribution of charge on irregular shaped conductors :

Potential at each point is equal.

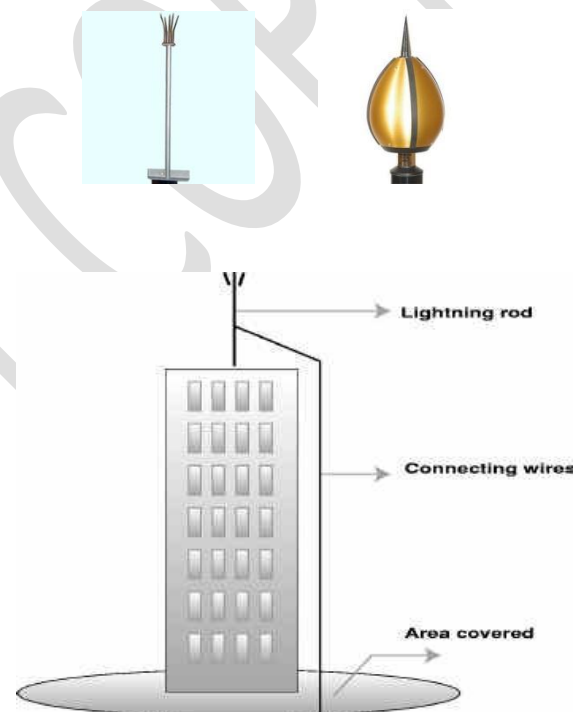
Electric field is always normal to surface.

Charge is distributed **unevenly**. Charge per unit area is more at the surface which has smaller radius. Therefore charge density is always more on the corners.



Corona discharge : when an uncharged body is brought near a charged body having sharp corners there is large number of charges at the corners. Due to induction, they induce large number of opposite charges. This creates a very strong Electric field between them. Finally the dielectric strength breaks-down and there is fast flow of charges. This Spray of charges by spiked object is called Corona discharge.

The lightning arrester work on the principles of Corona discharge where the charge pass through conductor of arrester, and the buildings are saved



Van-de-Graff generator

Introduction : It's a device used to create very high potential which is used for experiments of nuclear physics in which a charged particle with very high energy is required to hit the nucleus as target.

Principles : The following principles are involved in the device.

1. Charge on a conductor always move to and stay on the outer surface.
2. Pointed Corners conduct charges very effectively. (corona discharge)

3.If charge q is given to a body, its potential increases by relation $V = \frac{q}{4\pi\epsilon_0 r}$

4. If a body of small potential v' is placed inside a shell having potential V , then the body acquires potential $V+v'$

Description : There is a large spherical conducting shell of diameter of few meters placed on a non-conducting concrete structure few meters above the ground.

A long belt of insulating material like silk rubber or rayon moves around two pulleys, driven by a motor.

Two combs with pointed heads near belt are fitted. Lower one is spray comb and the upper Collecting Comb. The spray comb is connected with a high tension source.

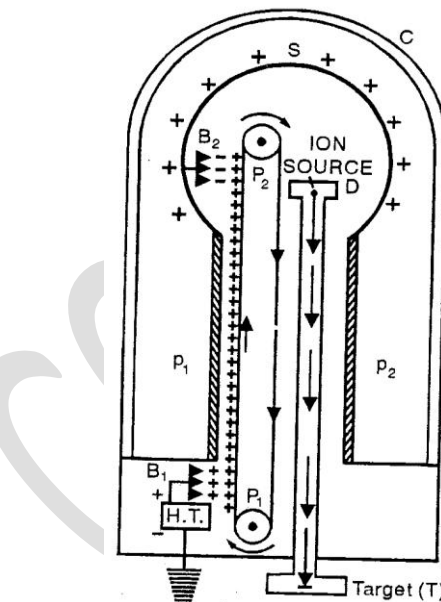
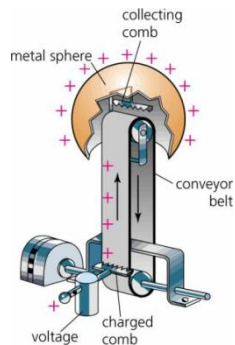
There is a discharge tube. One end having source of ion to be accelerated is inside the shell. Target is placed at the other end connected to earth.

The whole system is enclosed in a steel chamber filled with nitrogen or methane at high pressure.

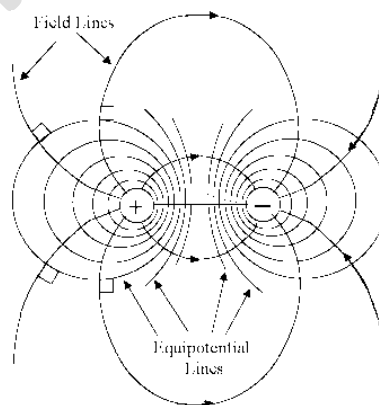
Working : The spray comb is given a positive potential ($\approx 10^4$ Volt) w.r.t. earth by the source of high Tension. Due to sharp points there is spray of charge on belt. The belt moves up with power of motor. When the charges reach near upper comb, due to induction and corona discharge the charge on belt is transferred to comb. From comb it moves to inner layer of shell. Since charge always stay at the outer surface, it moves to outer surface and the inner surface again become without any charge, ready to receive fresh charge again. As shell receive charge it Potential increase according to relation $V = \frac{q}{4\pi\epsilon_0 r}$. This potential is distributed all over and inside the shell.

The new charged particles which are coming having small potential v' from lower comb, acquire potential $V+v'$ due to their position inside the shell. There new potential is slightly higher than shell, therefore charges move from belt to comb to shell. This increases V further. This process keeps on repeating and V increase to a very high value, that is break-down voltage of compressed nitrogen $\approx 10^7$ volt.

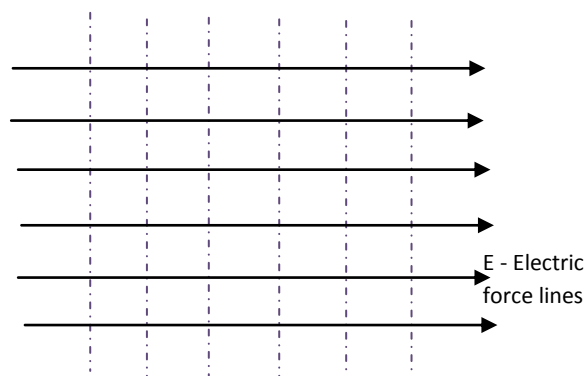
The ion inside discharged plate also acquires this potential due to its location inside the shell. Its energy increases by relation $U = qV$. The target is connected to earth at zero potential. Hence this ion gets accelerated and hits the target with very high energy.



Relation between Equipotential surfaces and E-Lines



Equipotential lines



CAPACITOR

It is a device to store charge and in turn store the electrical energy.

Any conductor can store charge to some extent. But we cannot give infinite charge to a conductor. When charge is given to a conductor its potential increases. But charge cannot escape the conductor because air, or medium around conductor is di-electric.

When due to increasing charge the potential increase to such extent that air touching the conductor starts getting ionized and hence charge gets leaked. No more charge can be stored and no more potential increase. This is limit of charging a conductor.

The electric field which can ionize air is $3 \times 10^9 \text{ vm}^{-1}$.

CAPACITANCE OF A CONDUCTOR

Term capacitance of a conductor is the ratio of charge to it by rise in its Potential

$$C = \frac{q}{V}$$

In this relation if $V=1$ then $C=q$. Therefore ,

Capacitance of a conductor is equal to the charge which can change its potential by one volt.

Unit of capacitance : Unit of capacitance is farad, (symbol F).

One farad is capacitance of such a conductor whose potential increase by one volt when charge of one coulomb is given to it.

One coulomb is a very large unit. The practical smaller units are

i. Micro farad (μF) = 10^{-6}F . (used in electrical circuits)

ii Pieco farad (pF) = 10^{-12} used in electronics circuits

Expression for capacitance of a spherical conductor :

If charge q is given to a spherical conductor of radius r , its potential rise by $V = \frac{q}{4\pi\epsilon_0 r}$

Therefore capacitance $C = \frac{q}{V} = q / \frac{q}{4\pi\epsilon_0 r} = 4\pi\epsilon_0 r$

Or for a sphere $C = 4\pi\epsilon_0 r$

The capacitor depends only on the radius or size of the conductor.

The capacitance of earth (radius 6400 km) is calculated to be 711×10^6 coulomb.

PARALLEL PLATE CAPACITOR : -

Since single conductor capacitor do not have large capacitance , parallel plate capacitors are constructed.

Principle : Principle of a parallel plate capacitor is that an uncharged plate brought bear a charged plate decrease the potential of charged plate and hence its capacitance ($C = \frac{q}{V}$) increase. Now it can take more charge. Now if uncharged conductor is earthed, the potential of charged plate further decreases and capacitance further increases. This arrangement of two parallel plates is called parallel plate capacitor.

Expression for capacitance :

Charge q is given to a plate

Of area 'A'. Another plate is kept at a distance 'd'.

After induction an

Electric field E is set-up

Between the plates. Here

$$q = \sigma A \quad \text{and} \quad E = \frac{\sigma}{\epsilon_0}$$

The Potential difference between plates is given by

$$V = Ed = \frac{\sigma}{\epsilon_0} d$$

$$\text{Now } C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma d}{\epsilon_0}} = \frac{\epsilon_0 A}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$

If a dielectric of dielectric constant K is inserted between the plates, then capacitance increase by factor K and become

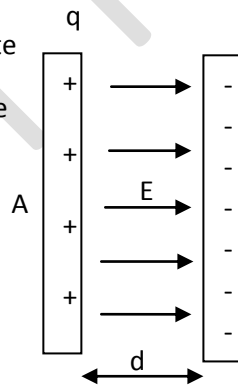
$$C = \frac{\epsilon_0 K A}{d}$$

Note : The capacitance depends only on its configuration i.e. plate area and distance, and on the medium between them.

The other examples of parallel plate capacitors is

$$\text{Cylindrical capacitor } C = \frac{4\pi\epsilon_0 KL}{\log r^2/r_1}$$

$$\text{and Spherical capacitor. } C = \frac{4\pi\epsilon_0 K r_2 r_1}{\log r_2 - r_1}$$

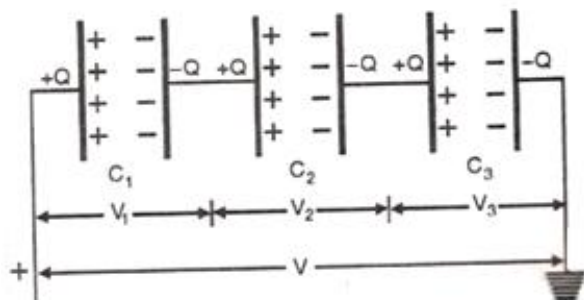


Combination of capacitors

Capacitors can be combined in two ways. 1. Series and 2. Parallel.

Series Combination :

If capacitors are connected in such a way that we can proceed from one point to other by only one path passing through all capacitors then all these capacitors are said to be in series.



Here three capacitors are connected in series and are connected across a battery of P.D. 'V'.

The charge q given by battery deposits at first plate of first capacitor. Due to induction it attracts $-q$ on the opposite plate. The pairing $+ve$ q charges are repelled to first plate of second capacitor which in turn induces $-q$ on the opposite plate. Same action is repeated to all the capacitors and in this way all capacitors get q charge. As a result ; the charge given by battery q , every capacitor gets charge q .

The Potential Difference V of battery is sum of potentials across all capacitors. Therefore

$$V = v_1 + v_2 + v_3$$

$$v_1 = \frac{q_1}{c_1}, v_2 = \frac{q_2}{c_2}, v_3 = \frac{q_3}{c_3}$$

Equivalent Capacitance : The equivalent capacitance across the combination can be calculated as $C_e = q/V$

$$\begin{aligned} \text{Or } 1/C_e &= V/q \\ &= (v_1 + v_2 + v_3) / q \\ &= v_1/q + v_2/q + v_3/q \end{aligned}$$

$$\text{Or } 1/C_e = 1/C_1 + 1/C_2 + 1/C_3$$

The equivalent capacitance in series decreases and becomes smaller than the smallest member.

In series q is same. Therefore by $q=cv$, we have

$$c_1 v_1 = c_2 v_2 = c_3 v_3$$

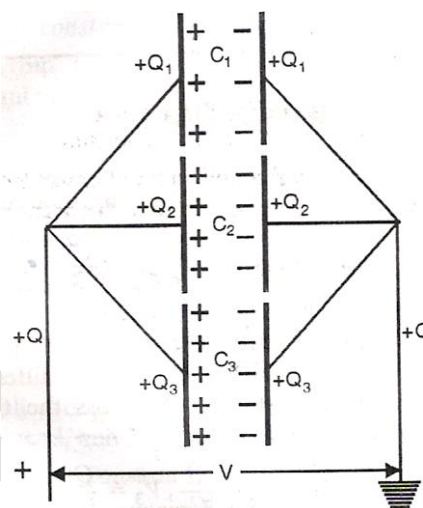
or $v \propto \frac{1}{c}$ i.e. larger c has smaller v , and smaller c has larger v across it.

$$\text{For 2 capacitor system } C = \frac{c_1 c_2}{c_1 + c_2}, \text{ and } v_1 = \frac{c_2}{c_1 + c_2} \cdot v$$

If n capacitors of capacitance c are joined in series then equivalent capacitance $C_e = \frac{c}{n}$

Parallel combination :

If capacitors are connected in such a way that there are many paths to go from one point to other. All these paths are parallel and the capacitance of each path is said to be connected in parallel.



Here three capacitors are connected in parallel and are connected across a battery of P.D. 'V'.

The potential difference across each capacitor is equal and it is same as P.D. across the battery.

The charge given by source is divided and each capacitor gets some charge. The total charge

$$q = q_1 + q_2 + q_3$$

Each capacitor has charge

$$q_1 = C_1 V, q_2 = C_2 V, q_3 = C_3 V$$

Equivalent Capacitance : We know that

$$q = q_1 + q_2 + q_3$$

$$\text{divide by } v \quad \frac{q}{v} = \frac{q_1}{v} + \frac{q_2}{v} + \frac{q_3}{v}$$

$$\text{or, } C = C_1 + C_2 + C_3$$

The equivalent capacitance in parallel increases, and it is more than the largest in parallel.

In parallel combination V is same therefore

$$(v =) \quad \frac{q_1}{c_1} = \frac{q_2}{c_2} = \frac{q_3}{c_3}$$

In parallel combination $q \propto c$. Larger capacitance larger is charge.

Charge distribution : $q_1 = C_1 V, q_2 = C_2 V, q_3 = C_3 V.$

In 2 capacitor system charge on one capacitor

$$q_1 = \frac{c_1}{c_1 + c_2} \cdot q$$

n capacitors in parallel give $C = nc$

Energy stored in a capacitor: When charge is added to a capacitor then charge already present on the plate repel any new incoming charge. Hence a new charge has to be sent by applying force and doing work on it. All this work done on charges become energy stored in the capacitor.

At any instant work done $dw = V \cdot dq$, or $dw = \frac{q}{c} \cdot dq$

Therefore work done in charging

the capacitor from charge 0 to q $W = \int_0^q \frac{q}{c} \cdot dq$
 $= \frac{1}{c} \int_0^q q \cdot dq = \frac{1}{c} \frac{q^2}{2} = \frac{q^2}{2c}$

This work done convert into electrical Potential

Energy stored in the capacitor $U = \frac{1}{2} \frac{q^2}{c} = \frac{1}{2} qv = \frac{1}{2} cv^2$

This energy is stored in the form of Electric field between the plates.

Energy per unit volume $u = \frac{1}{2} cv^2/V = \frac{1}{2} \frac{\epsilon_0 AE^2 d^2}{dAd}$

Or, energy density $u = \frac{1}{2} \epsilon_0 E^2$

Connecting two charged capacitors :- When two conductors are connected the charges flow from higher potential plate to lower potential plate till they reach a common potential.

Common Potential : A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2 . The charge flow from higher potential to lower potential and it reach an in between value V such that

$V = \frac{\text{total charge}}{\text{Total capacitance}}$ or $V = \frac{c_1 v_1 + c_2 v_2}{c_1 + c_2}$

Loss of Energy on connecting two conductors :

A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2 . The charge flow from higher potential to lower potential and in this process it loses some energy as charge has to do some work while passing through connecting wire. The energy is lost in form of heat of connecting wire.

Expression for energy lost : In the above two capacitors the energy contained in the two before connection, $E_1 = \frac{1}{2} c_1 v_1^2 + \frac{1}{2} c_2 v_2^2 \dots \dots \dots (i)$

Common Potential after connection, $V = \frac{c_1 v_1 + c_2 v_2}{c_1 + c_2}$

Combined capacitance $c_1 + c_2$

Energy in combination : $\frac{1}{2} (c_1 + c_2) \left(\frac{c_1 v_1 + c_2 v_2}{c_1 + c_2} \right)^2$

Hence Loss in energy : $E_1 - E_2$

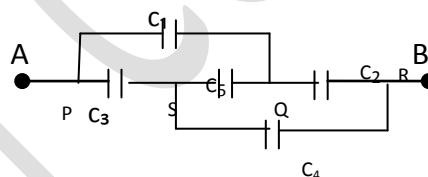
$= \left\{ \frac{1}{2} c_1 v_1^2 + \frac{1}{2} c_2 v_2^2 \right\} - \left\{ \frac{1}{2} (c_1 + c_2) \left(\frac{c_1 v_1 + c_2 v_2}{c_1 + c_2} \right)^2 \right\}$
 $= \frac{1}{2} \left(\frac{c_1 c_2}{c_1 + c_2} \right) (v_1 - v_2)^2$

It is a positive number which confirm that there is loss of energy in transfer of charges. Hence

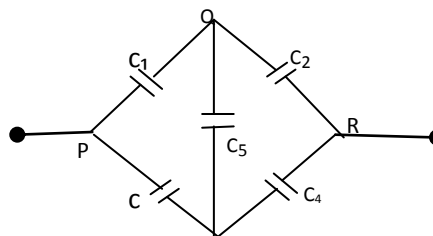
loss of energy $= \frac{1}{2} \left(\frac{c_1 c_2}{c_1 + c_2} \right) (v_1 - v_2)^2$

Wheatstone bridge in combination of capacitors :

Five capacitors joined in following manner is called wheatstone bridge connection.



Or, it is redrawn as under :



In the above arrangement, if ratio $c_1/c_2 = c_3/c_4$ then the bridge is said to be *balanced*. In such case the potential at point Q and S are equal.

The potential across c_5 is zero hence it does not carry any charge. In this way it is not participating in storage of charges. Then it can be omitted for further calculations. Calculations are done for c_1, c_2, c_3 and c_4 only.

Dielectrics: are non conducting materials. They do not have free charged particles like conductors have. They are two types.

- i. Polar : The centre of +ve and -ve charges do not coincide. Example HCl, H₂O, They have their own dipole moment.

- ii. Non-Polar : The centers of +ve and -ve charges coincide. Example CO_2 , C_6H_6 . They do not have their own dipole moment.

In both cases, when a dielectric slab is exposed to an electric field, the two charges experience force in opposite directions. The molecules get elongated and develops i. surface charge density σ_p and not the volumetric charge density. This leads to development of an induced electric field \mathbf{E}_p , which is in opposition direction of external electric field \mathbf{E}_0 . Then net electric field \mathbf{E} is given by $\mathbf{E} = \mathbf{E}_0 - \mathbf{E}_p$. This indicates that net electric field is decreased when dielectric is introduced.

The ratio $\frac{E_0}{E} = K$ is called **dielectric constant** of the dielectric.

Clearly electric field inside a dielectric is $E = \frac{E_0}{K}$.

Dielectric polarization : when external electric field E_0 is applied, molecules get polarized and this induced dipole moment of an atom or molecule is proportionate to applied electric field. i.e. $p \propto E_0$

$$\text{or } p = \alpha \epsilon_0 E_0$$

here α is a constant called atomic / molecular polarizability. It has dimensions of volume (L^3) it has the order of 10^{-29} to 10^{-30} m^3 .

This **polarization** is a vector quantity and is related to resultant electric field \mathbf{E} as under :

$$\vec{p} = \chi_e \epsilon E$$

Where χ_e is a constant called electric susceptibility of the dielectric.

The induced charge σ_p is due to this polarization, hence

$$\sigma_p = \vec{p} \cdot \hat{n}$$

When this dielectric is introduced between the two plates having charge density σ then resultant electric field can be related as

$$\begin{aligned} E \cdot \hat{n} = E - E_p &= \frac{\sigma - \sigma_p}{\epsilon_0} = \frac{\sigma - \vec{p} \cdot \hat{n}}{\epsilon_0} \\ \text{or } (\epsilon^0 E + p) \cdot \hat{n} &= \sigma \\ \text{or } \vec{D} \cdot \hat{n} &= \sigma \end{aligned}$$

The quantity \vec{D} is called **electric displacement** in dielectric.

We can prove that $K = 1 + \chi_e$